Fédération de recherche André Marie Ampère

ENS DE LYON

# Cours du Collège de France 2011-'12

Gabriele Veneziano

# De la corde hadronique à la gravitation quantique... et retour

Cours I&II (2 mars):

Rise and fall of the (fake) hadronic string

Cours III&IV (5 mars):

String theory and quantum gravity

Cours V&VI (12 mars):

From gravity back to the (true?) hadronic string

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# De la corde hadronique à la gravitation quantique... et retour

## Cours I : 2 mars 2012

Une Brève Histoire de la Corde Hadronique

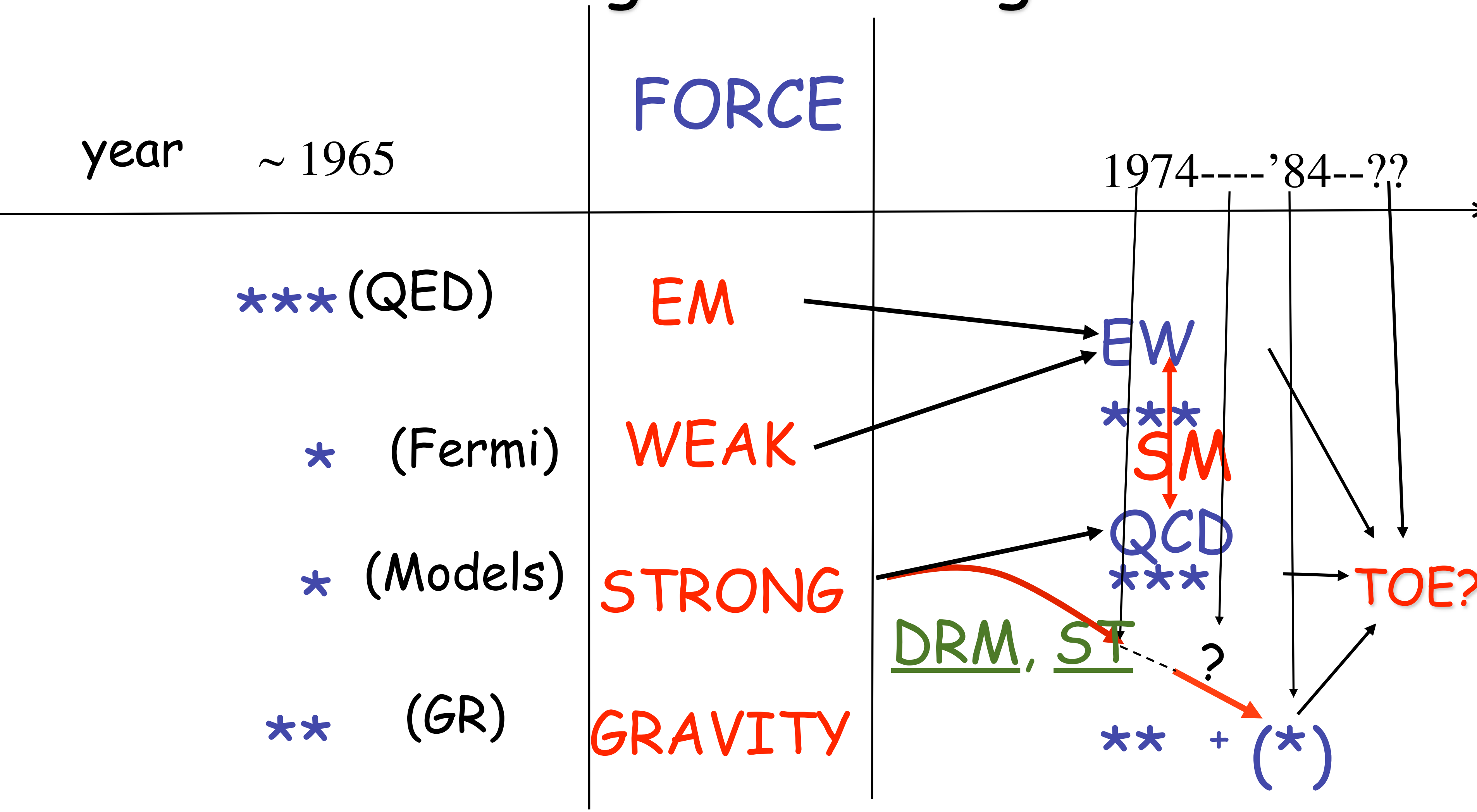
ou

Rise and Fall of the Hadronic String

# A BOTTOM UP APPROACH

(quite unlike what is usually done)

# Situating the birth of string theory within an evolving "Michelin guide"



# STRONG INTERACTIONS in the 60s

No Theory, rather:

A handful of models capturing one or another aspect of hadronic physics e.g.

- Short range i.e. no massless particles
- Symmetries, conservation laws (P, C, T, I, SU(3),…)
- Many metastable states (resonances) extending to large J: an ever increasing zoo?

# Why did we take the (a posteriori) wrong way?

A QFT approach looked hopeless:

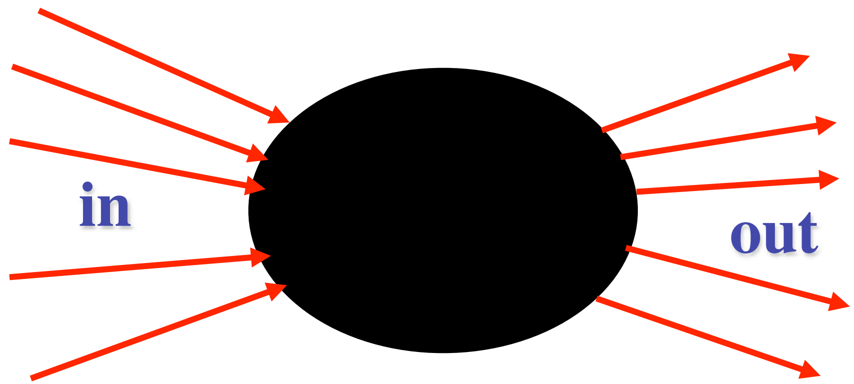
1. Too many d.o.f. => too many fields
2. High-J QFT's are pathological (J=2 is already bad-enough!)

An S-matrix approach looked more promising

# The S-Matrix (Heisenberg 1943)

$$\langle out|S|in\rangle = S(in \to out) = complex\ number$$

$$|S(in \to out)|^2 = Prob.\ for: in \to out$$




- **Symmetries:** easy to implement on S
- **Causality** => analyticity, dispersion relations
- **Conservation of Prob** => unitarity constraint: $SS^\dagger = 1$

# Organizing the hadronic zoo

A) Group theory:

- $SU(2)_I$ , $SU(3)_F$, same-J particles
- SU(4), SU(6) ... combining $\Delta J \leq 1$ particles (not rigorous)

B) Regge theory of complex J

- For combining different-J particles (Regge)
- For describing high-energy scattering (Chew-Mandelstam)

# Sketch of Regge's theory of complex J

- Consider non-relativistic potential scattering. Expand the scattering amplitude (~ the S-matrix) in partial waves:

$$A(E,\theta) = \sum_{J=0}^{\infty} A_J(E) P_J(cos\theta)$$

- In 1959 Tullio Regge had the bold idea of looking at $A_J(E)$ as an analytic function of complex J. He found that, quite generically, there were poles in J at J $=\alpha(E)$:

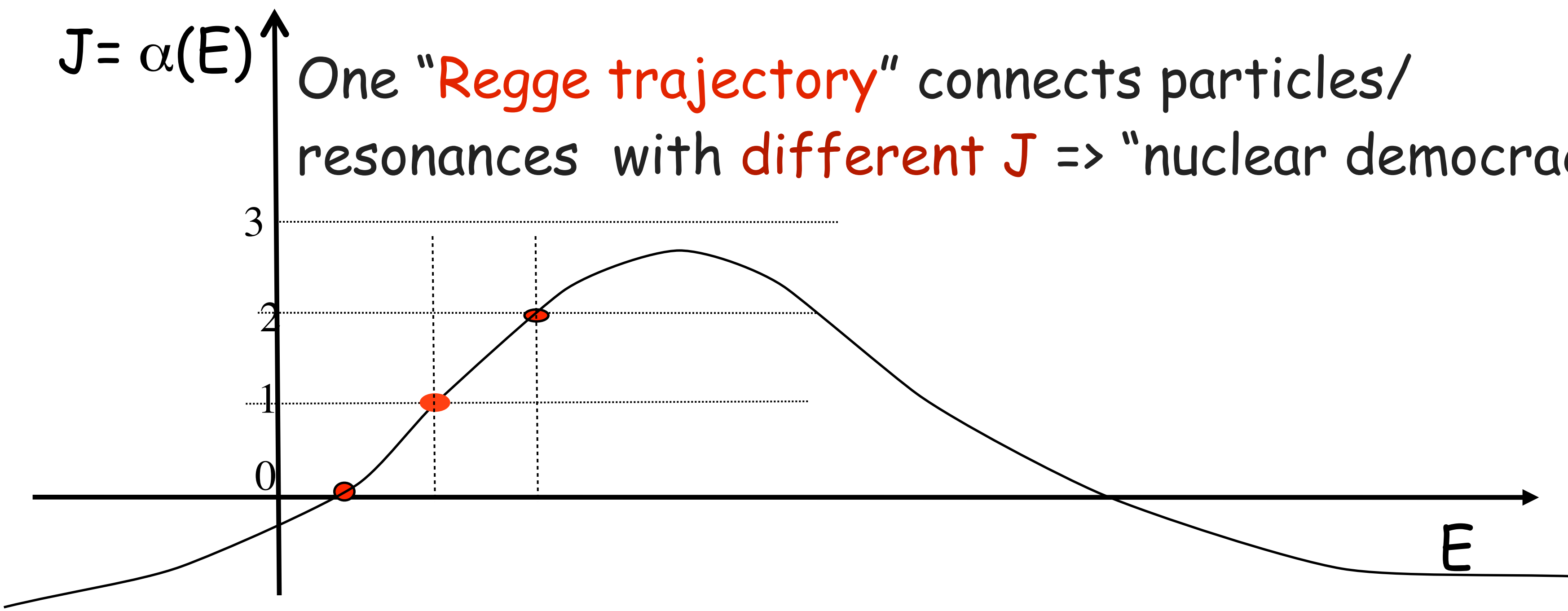
$$A_J(E) \sim \frac{\beta(E)}{J-\alpha(E)}$$

$$A_J(E) \sim \frac{\beta(E)}{J-\alpha(E)}$$

$$\alpha(E_n) = n \Rightarrow A(E,\theta) = \frac{\beta(E_n)}{n-\alpha(E)} P_n(cos\theta) \sim -\frac{\beta(E_n)}{\alpha'(E-E_n)} P_n(cos\theta)$$

This is just the contribution to the scattering amplitude of a single resonance of energy $E_n$.

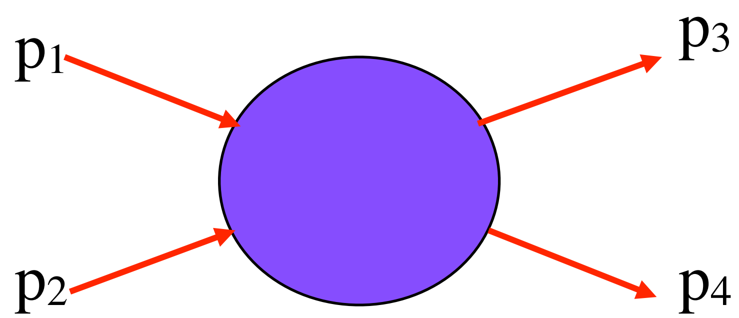
One "Regge trajectory" connects particles/ resonances with different J => "nuclear democracy".



# Chew-Mandelstam application of Regge theory in relativistic scattering

Relativistic 2-body scattering amplitude A(s,t) expanded in t-channel partial waves:

$$\begin{aligned} s = -(p_1+p_2)^2 &= -(p_3+p_4)^2 \\ t = -(p_1-p_3)^2 &= -(p_2-p_4)^2 \\ u = -(p_1-p_4)^2 &= -(p_2-p_3)^2 \\ s+t+u &= \sum m_i^2 \end{aligned}$$




s,t, u are the so-called Mandelstam variables

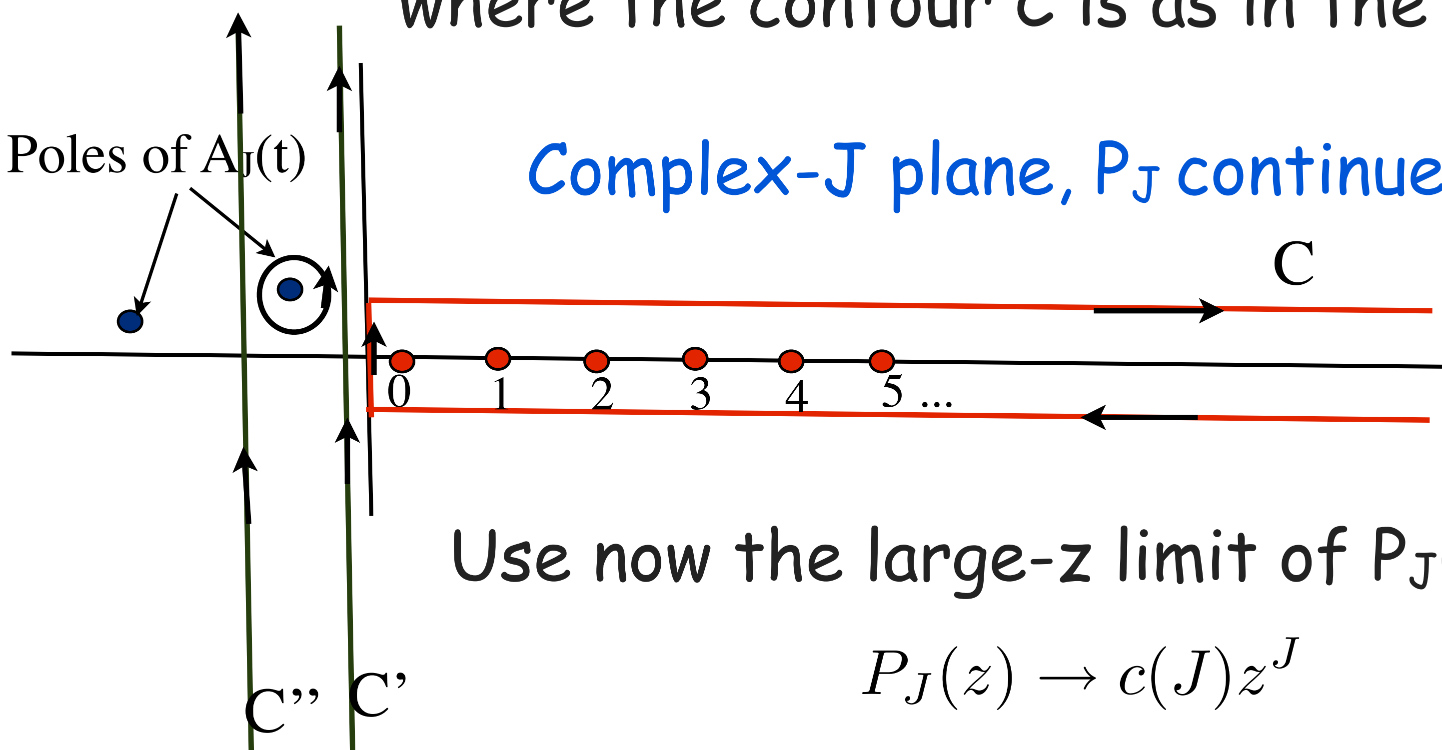
$$A(s,t) = \sum_{J=0}^{\infty} A_J(t) P_J(cos\theta_t) \quad ; \quad cos\theta_t = 1 + 2s/t$$

considered in the "unphysical" region: s large and positive, t < 0 fixed. $cos\theta_s = 1 + 2t/s \rightarrow 1$

# The sum diverges but can be analytically continued using a trick due to Froissart & Gribov

$$A(s,t) = \sum_{J=0}^{\infty} A_J(t) P_J(cos\theta_t) = \frac{1}{2i} \int_C dJ \, \frac{e^{i\pi J}}{sin(\pi J)} A_J(t) P_J(cos\theta_t)$$

where the contour $C$ is as in the figure.



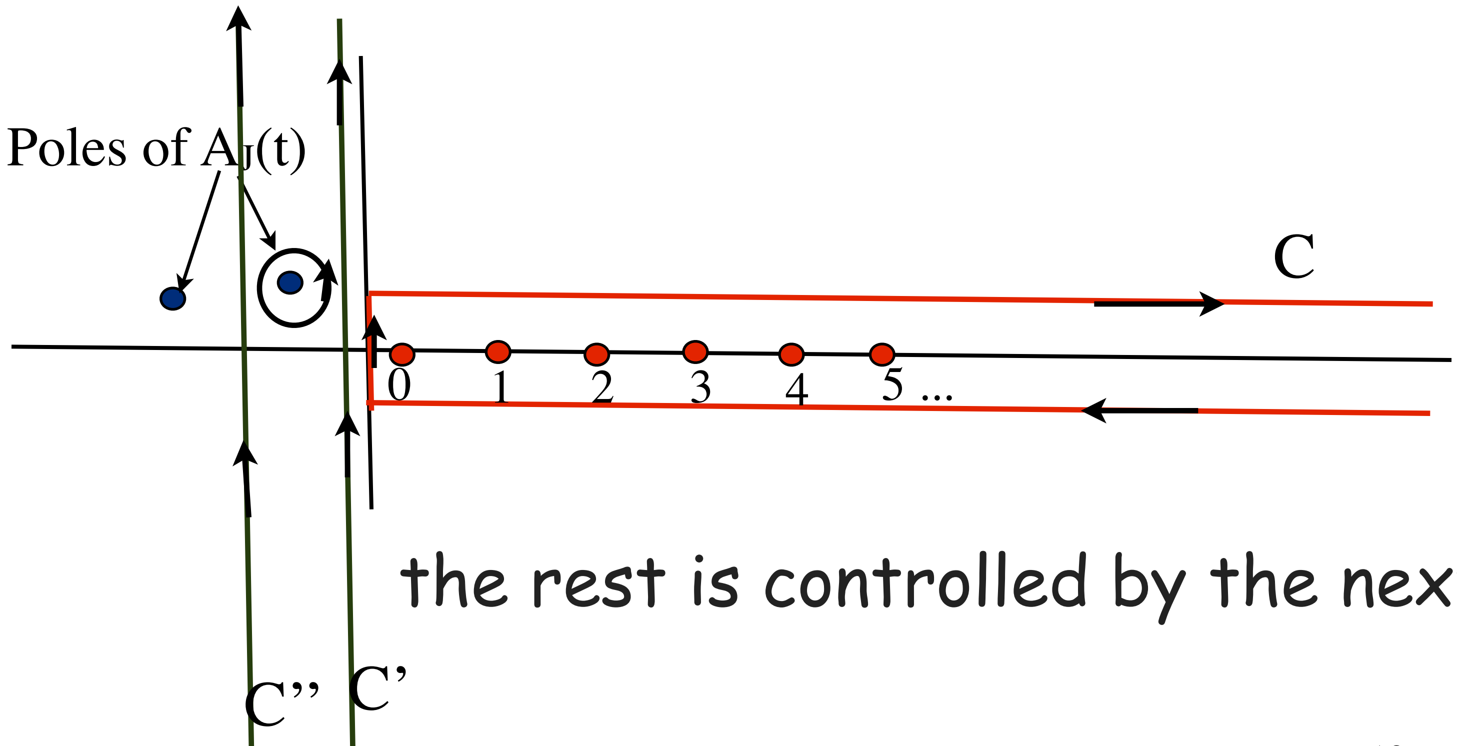

Complex-J plane, $P_J$ continued as well

Use now the large-z limit of $P_J(z)$:

$$P_J(z) \rightarrow c(J) z^J$$

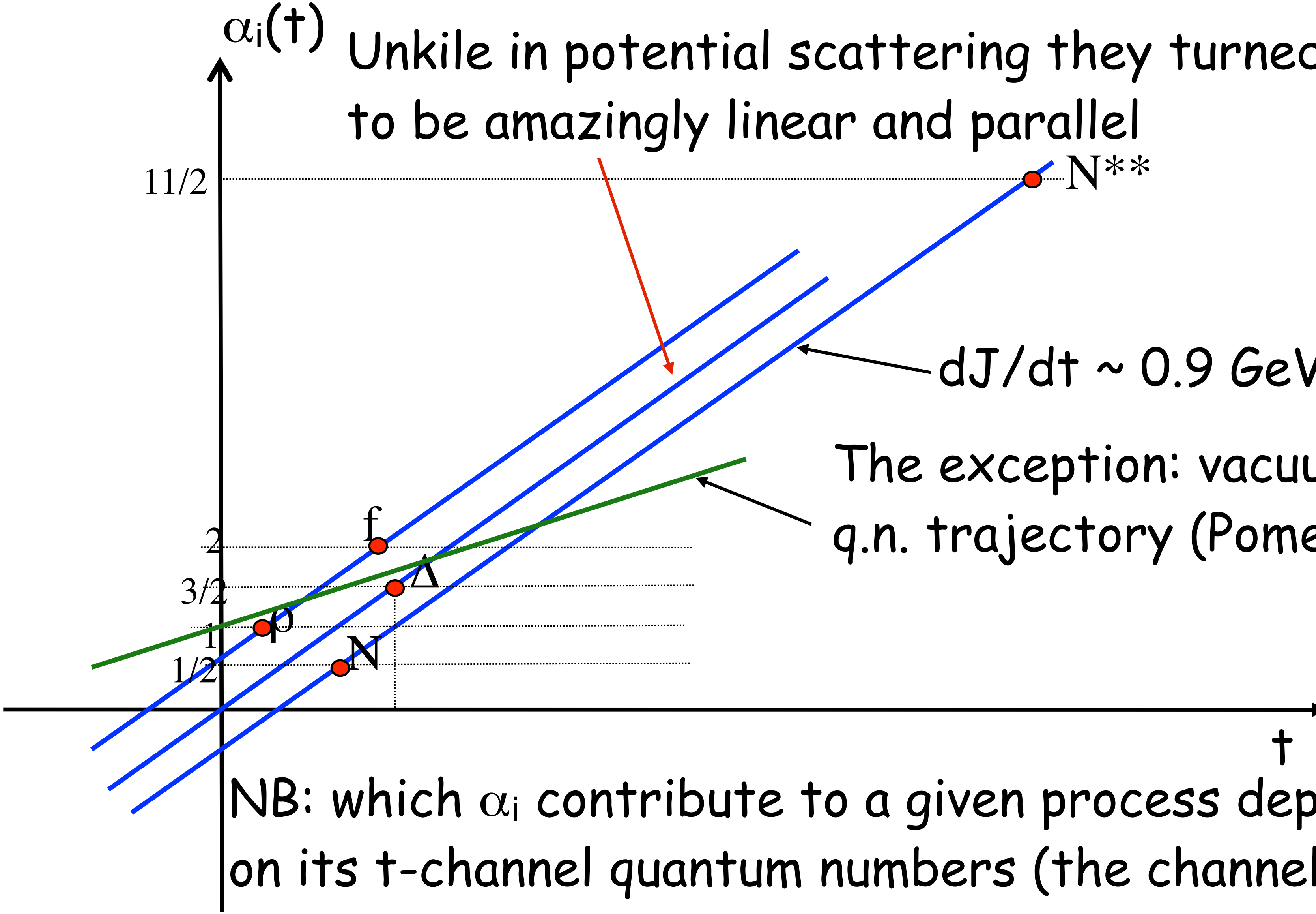
Deforming the contour from C to C' to C'' (which includes the little circle around the rightmost Regge pole) we get, from the latter:

$$A(s,t) \sim \frac{\beta(t)}{sin(\pi\alpha(t))}\left[(-s)^{\alpha(t)} \pm (-u)^{\alpha(t)}\right] \sim \frac{\beta(t)[e^{i\pi\alpha} \pm 1]}{sin(\pi\alpha(t))} s^{\alpha(t)}$$




the rest is controlled by the next pole..

Unkile in potential scattering they turned out to be amazingly linear and parallel




dJ/dt ~ 0.9 $GeV^{-2}$

The exception: vacuum q.n. trajectory (Pomeron)

NB: which $\alpha_i$ contribute to a given process depends on its t-channel quantum numbers (the channel whose Mandelstam variable is kept fixed )

# Chew's "expensive" bootstrap…

Add to the general constraints of symmetry, causality, unitarity that of Nuclear Democracy

"All hadrons lie on Regge trajectories @ $M^2>0$;

All asymptotics fixed by same trajectories @ $M^2<0$"

Will this give a unique S-matrix?

A posteriori, Chew's program was too optimistic. We now believe the answer to the question to be negative.

String theory is a perfect example of Nuclear Democracy and satisfies the other constraints as well…but adds to them a crucial new dynamical input: strings!

# Superconvergence (S. Fubini ~ 1966)

Regge-Chew-Mandelstam theory can be combined with analyticity to get some interesting "sum rules"

The first example, superconvergence, applies when the t-channel quantum numbers are such that *A*(s,t) decreases at large s and fived t faster than 1/s. Writing a fixed-t (unsubtracted) dispersion relation for *A*,

$$A(s,t) = \frac{1}{\pi}\int ds' \frac{ImA(s',t)}{s'-s-i\epsilon}$$

and imposing that s*A*-->0 at large s we must have:

$$\int ds \; ImA(s,t) = 0$$

Inserting low-energy "data" met with very reasonable success

# Finite-energy sum rules (FESR)

In this case we use our theoretical (Regge) model at high energy and write a superconvergence relation for a subtracted amplitude: $A^{(sub)} = A(s,t) - A^{(R)}(s,t)$ so that $s\, A^{(sub)}$ goes to zero at large s. Limiting the integral to a finite value $s_0$ we get:

$$\int_0^{s_0} ds\ ImA(s,t) \sim \int_0^{s_0} ds\ ImA^{(R)}(s,t)$$

$s_0$ has to be taken judiciously. Using two such reasonable $s_0$

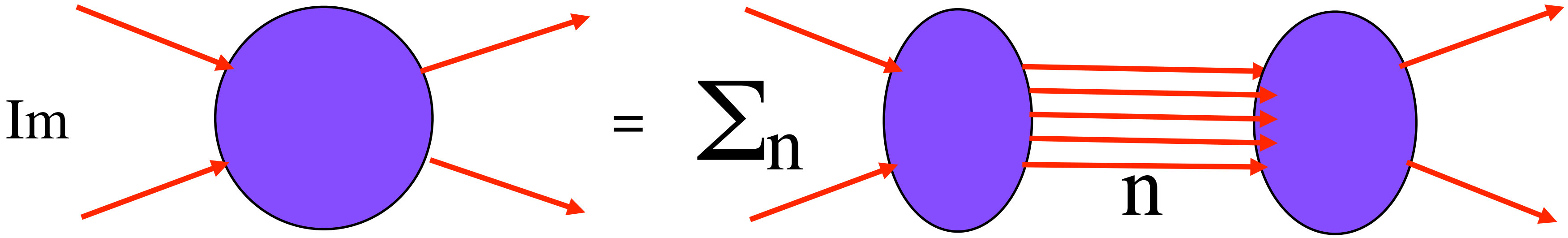
$$\int_{s_1}^{s_2} ds\ ImA(s,t) \sim \int_{s_1}^{s_2} ds\ ImA^{(R)}(s,t) = \sum \beta_i(t) \frac{s_2^{\alpha_i(t)+1} - s_1^{\alpha_i(t)+1}}{\alpha_i(t)+1}$$

Unitarity relates Im A to s-channel intermediate states hence we get a relation between s and t-channel quantities

The question is: what should we put on the s-channel (l.h.) side of the FESR?

Finding the correct answer to this question turned out to be one of the crucial steps towards the ultimate discovery of string theory...

Unitarity relates Im $A(s,t)$ to a sum over all the physical intermediate states that can appear in the s-channel with total c.o.m. energy $s^{1/2}$.

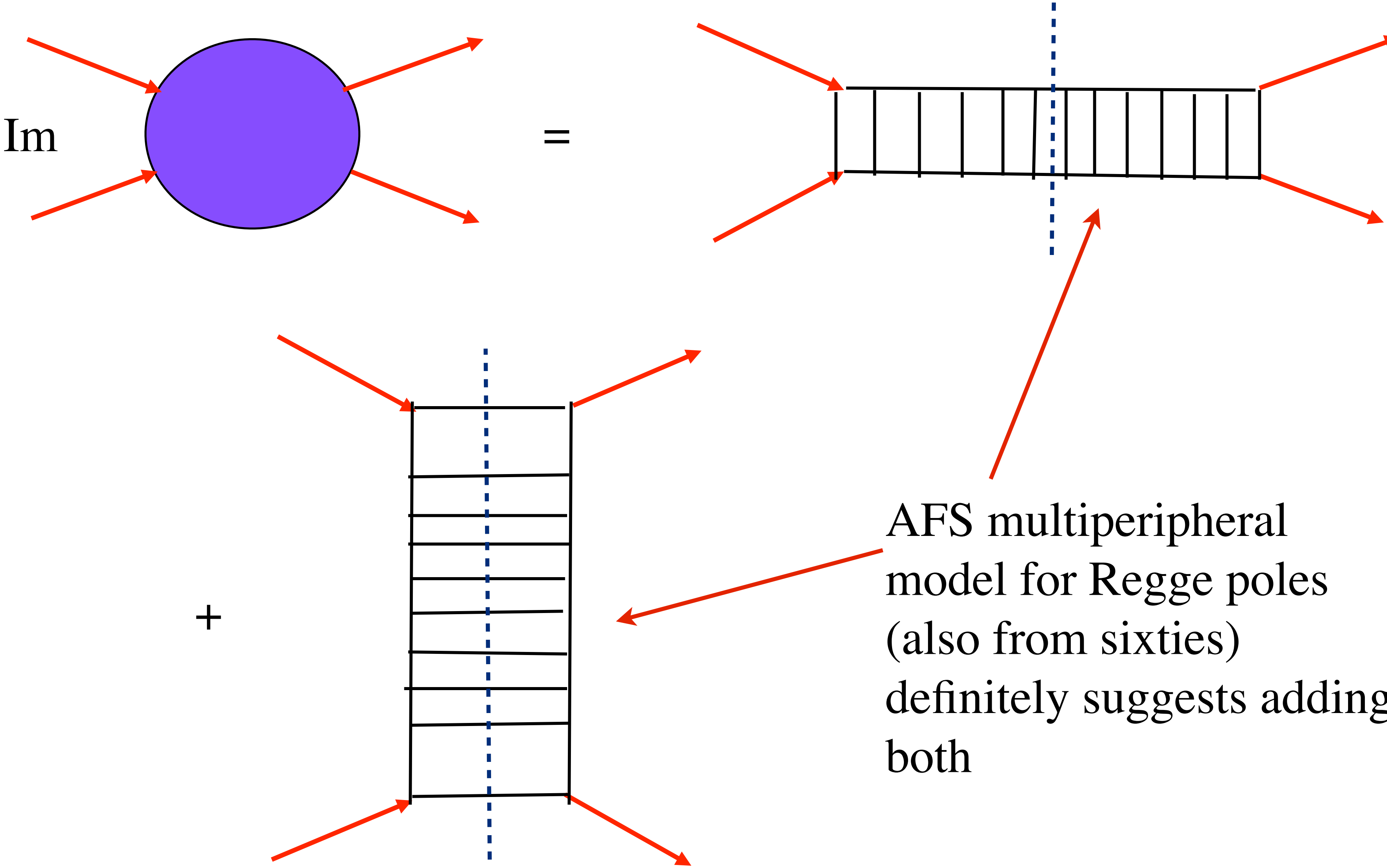



One obvious contribution was the one due to the resonances that could be produced in the s-channel (supposedly lying on that channel's Regge trajectories)

But what about other contributions that are making up the imaginary part of the t-channel Regge pole itself?

The prevailing belief at the time was that those two contributions had nothing to do with each other and that, therefore, should be added.

This was supported by QFT models for Regge poles

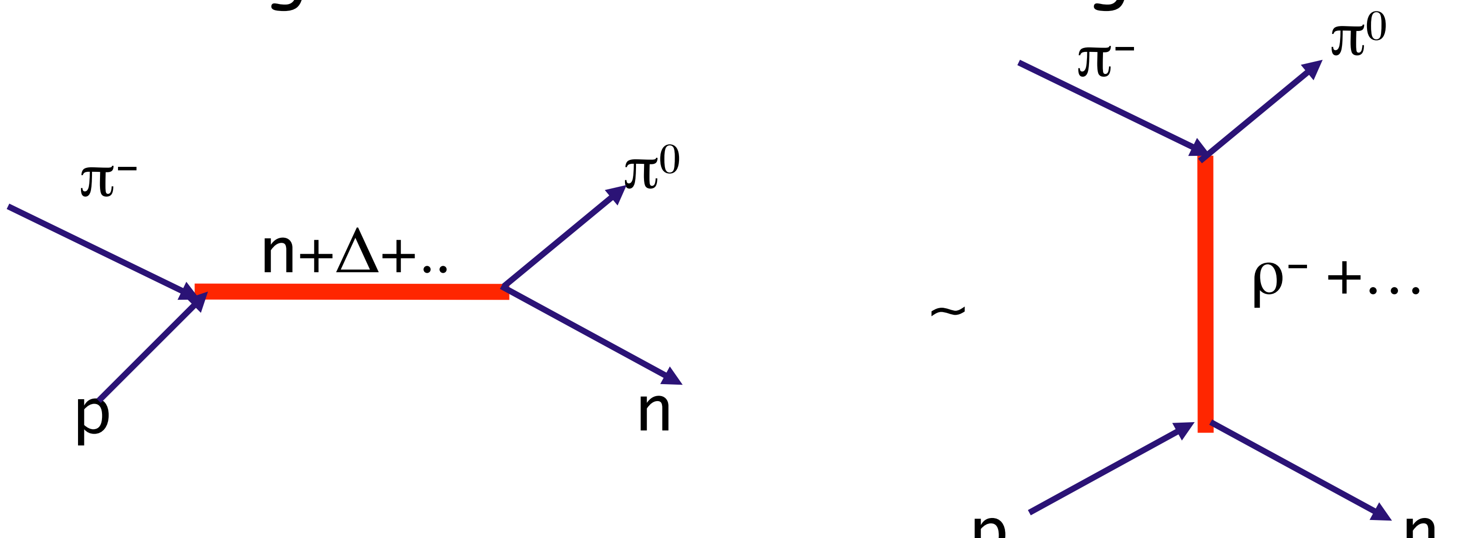

Im
=
+
AFS multiperipheral model for Regge poles (also from sixties) definitely suggests adding both

# DHS duality

Erice, 1967: Gell Mann bringing news from Caltech:

Dolen-Horn-Schmit duality: s-and t-channel descriptions are roughly equivalent, complementary, DUAL (Cf. QM's particle/wave duality)

Adding them = double counting!



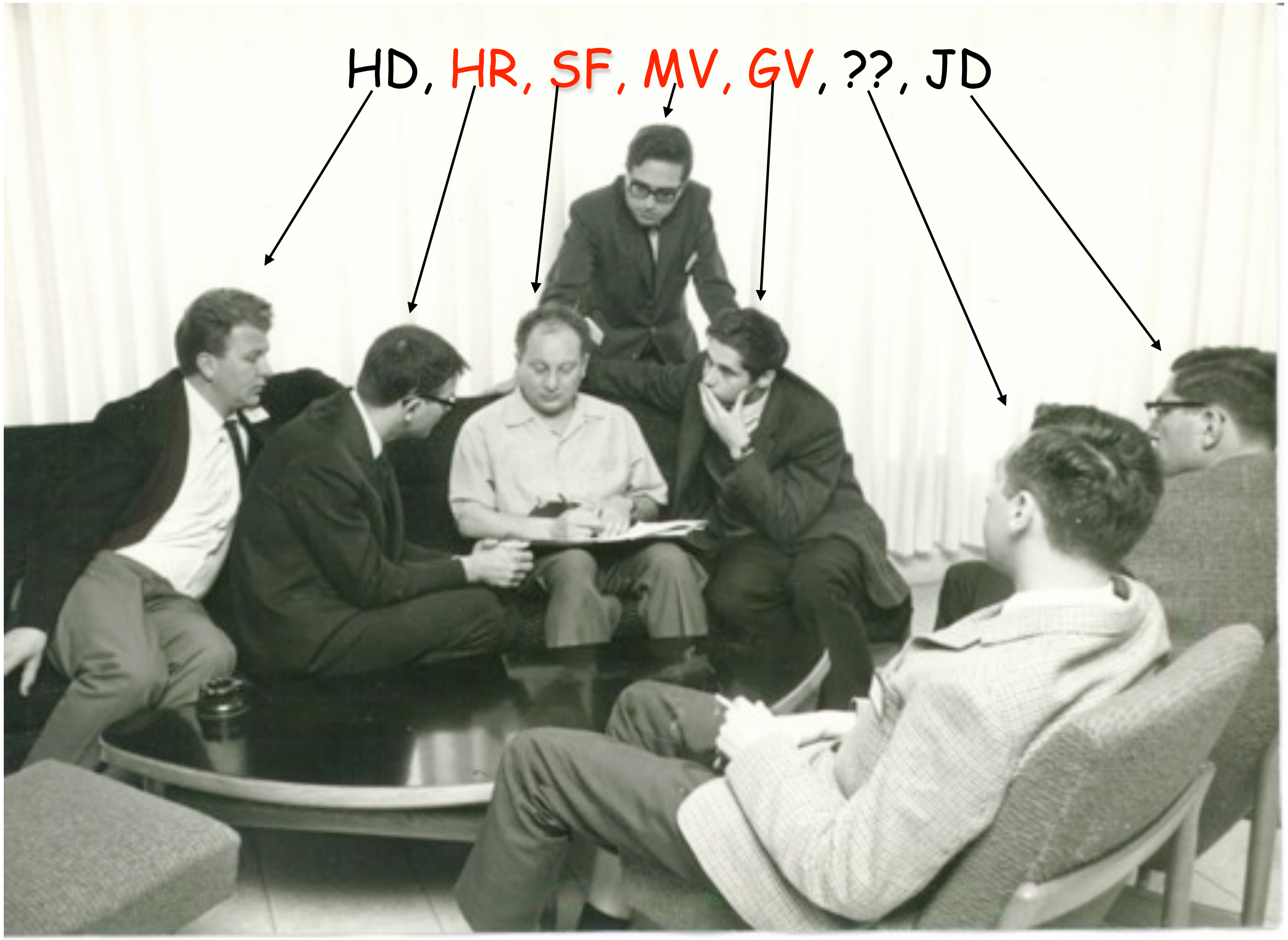
π N scattering looked too complicated
We* decided to consider a simpler case:

$\pi\, \pi \to \pi\, \omega$ (very symmetric & very selective in QN's)
($\rho$, $\rho^*$..)

Between the fall of 1967 and the summer of 1968 we made much progress in finding solutions to this "Easy Bootstrap".

*) Ademollo, Rubinstein, Virasoro, GV (+Bishari & Schwimmer) with much advice and encouragement by Sergio Fubini

# Weizmann Institute, 1967



# An exact (and simple) solution

The ARVV ansatz that worked amazingly well for the DHS bootstrap in $\pi\pi \rightarrow \pi\omega$ was simply*:

$$Im\,A(s,t) = \frac{\beta(t)}{\Gamma(\alpha(t))}(\alpha' s)^{\alpha(t)-1}(1+O(1/s))$$

with: $$\beta(t) \sim const.,\ \alpha(t) = \alpha_0 + \alpha' t$$

i.e. a linear leading Regge trajectory accompanied by parallel "daughter" trajectories. The latter, if suitably tuned, were improving the agreement in an increasingly large range of t

Which was the road that led from the above ansatz to an "exact solution"? Three main ingredients were used:

************************

*The extra 1/s is due to the non-zero helicity of $\omega$

1. Look at $A$ rather than at Im $A$ ($A$ = analytic function)
2. Impose exact crossing symmetry : $A(s,t) = A(t,s)$
3. Emphasize resonances over Regge ($A$ ~ meromorphic)

1. Easy to show that

$$Im\, A(s,t) = \frac{\beta(t)}{\Gamma(\alpha(t))}(\alpha' s)^{\alpha(t)-1}(1+O(1/s))$$ corresponds to :

$$\pi A(s,t) = \beta(t)\Gamma\left(1-\alpha(t)\right)\left(-\alpha' s\right)^{\alpha(t)-1}\left(1+O(1/s)\right)$$

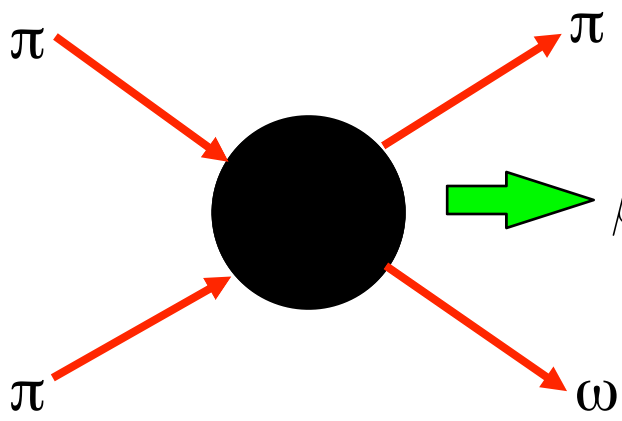
recalling: $$\Gamma(x)\Gamma(1-x) = \frac{\pi}{sin\pi x}$$

3. A(s,t) already exhibits resonances (poles) in the t-channel but only a smooth Regge behaviour in s: However, using

$$\frac{\Gamma(1-\alpha(s))}{\Gamma(2-\alpha(s)-\alpha(t))} \to (-\alpha' s)^{\alpha(t)-1}(1+O(1/s))$$

we can satisfy both 2. and 3. by simply writing:

$$\boxed{A(s,t) = \beta\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))} = \beta\, B(1-\alpha(s), 1-\alpha(t))}$$




Full x-ing symmetry then implies:

$$\begin{aligned}\beta^{-1}A(s,t) &= B\left(1-\alpha(s), 1-\alpha(t)\right)\\ &+ B\left(1-\alpha(u), 1-\alpha(t)\right)\\ &+ B\left(1-\alpha(s), 1-\alpha(u)\right)\end{aligned}$$

Exact DHS duality is implied by:
1. Analyticity (dispersion relations);
2. All singularities are poles corresponding to resonances
3. Good (Regge) asymptotics!
=> duality between two infinite sets of resonances in "dual" channels!

Properties best analyzed by using a well-known integral representation of the Beta-function:

$$B\left(1-\alpha(s), 1-\alpha(t)\right)=\int_0^1 dx\ x^{-\alpha(s)}(1-x)^{-\alpha(t)}$$

Analytic cont. needed: only converges for suff. negative s,t.

$$\beta(t) \sim const.,\ \alpha(t)=\alpha_0+\alpha' t$$

$$B\left(1-\alpha(s), 1-\alpha(t)\right) = \int_0^1 dx\; x^{-\alpha(s)}(1-x)^{-\alpha(t)}$$

1. Crossing symmetry (DHS duality): x --> (1-x)
2. All singularities are poles: e.g. expanding integrand in powers of x (or of (1-x)) gives s-t duality in a nicer form:

$$\sum_{n=0}^{\infty} \frac{C_n(t)}{s-m_n^2} = \sum_{n=0}^{\infty} \frac{C_n(s)}{t-m_n^2}$$

3. Good (Regge) asymptotics: as s becomes very large (with t fixed) integral is dominated by x ~ 1 region. Implies duality!

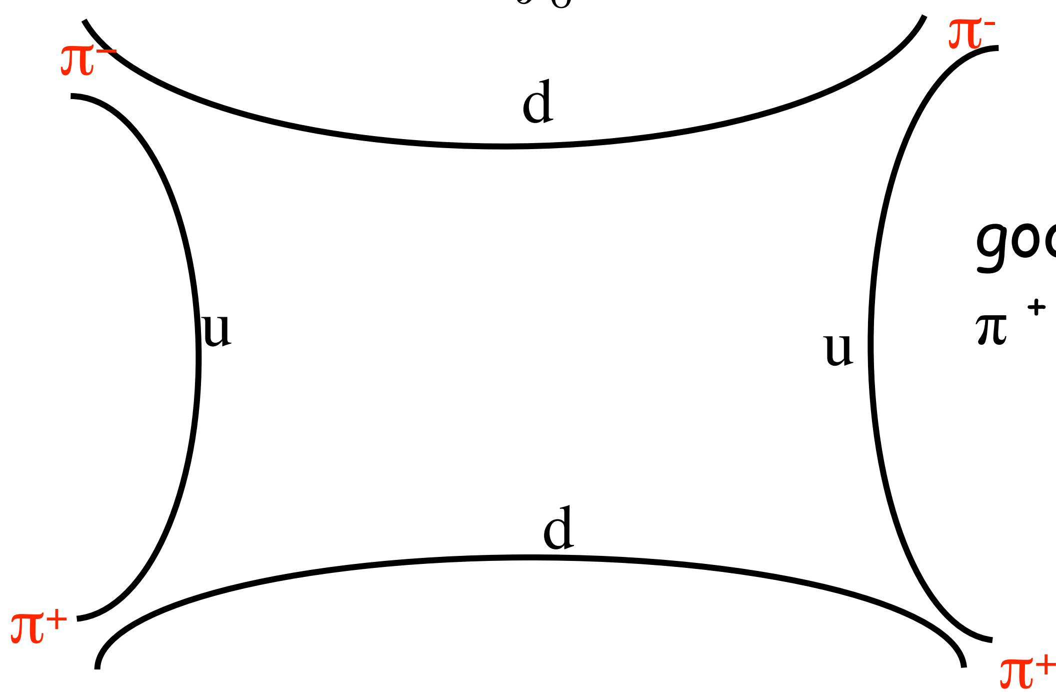
$$\int_0^1 dx\; (1-x)^{-\alpha_s} x^{-\alpha_t} \sim \int_0^{\cdots} dx\; e^{x\alpha_s} x^{-\alpha_t}$$

$$\sim (-\alpha_s)^{\alpha_t - 1} \int_0^{\infty} dz e^{-z} z^{-\alpha_t} = (-\alpha_s)^{\alpha_t-1}\Gamma(1-\alpha_t)$$

In the following our starting point will be the 2->2 scattering amplitude for spinless particles (e.g. ππ scattering). This is obtained by the simple replacement:

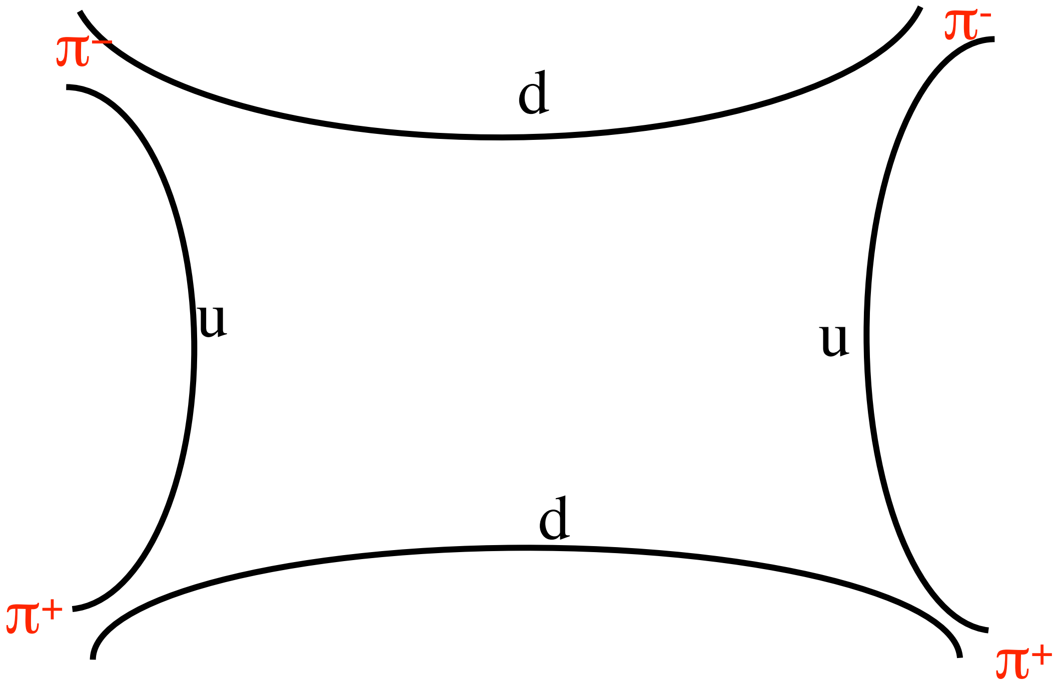
$$\frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))} \to \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$

$$= \quad B(-\alpha(s),-\alpha(t)) = \int_0^1 dx\; x^{-1-\alpha(s)}(1-x)^{-1-\alpha(t)}$$




good to describe $\pi^+\pi^-$ scattering?

A more successful (but still phenomenological) model was proposed by Lovelace giving rise to great hopes...




$$A(\pi^+\pi^- \to \pi^+\pi^-) = g^2 \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))}$$

$$\alpha(t) = \alpha_\rho(t) \sim 0.5 + 0.9t \text{ GeV}^{-2}$$

Another successful application was to use the original model to describe the (Dalitz plot for the) process $\omega \to 3\,\pi$

# Fear of ghosts

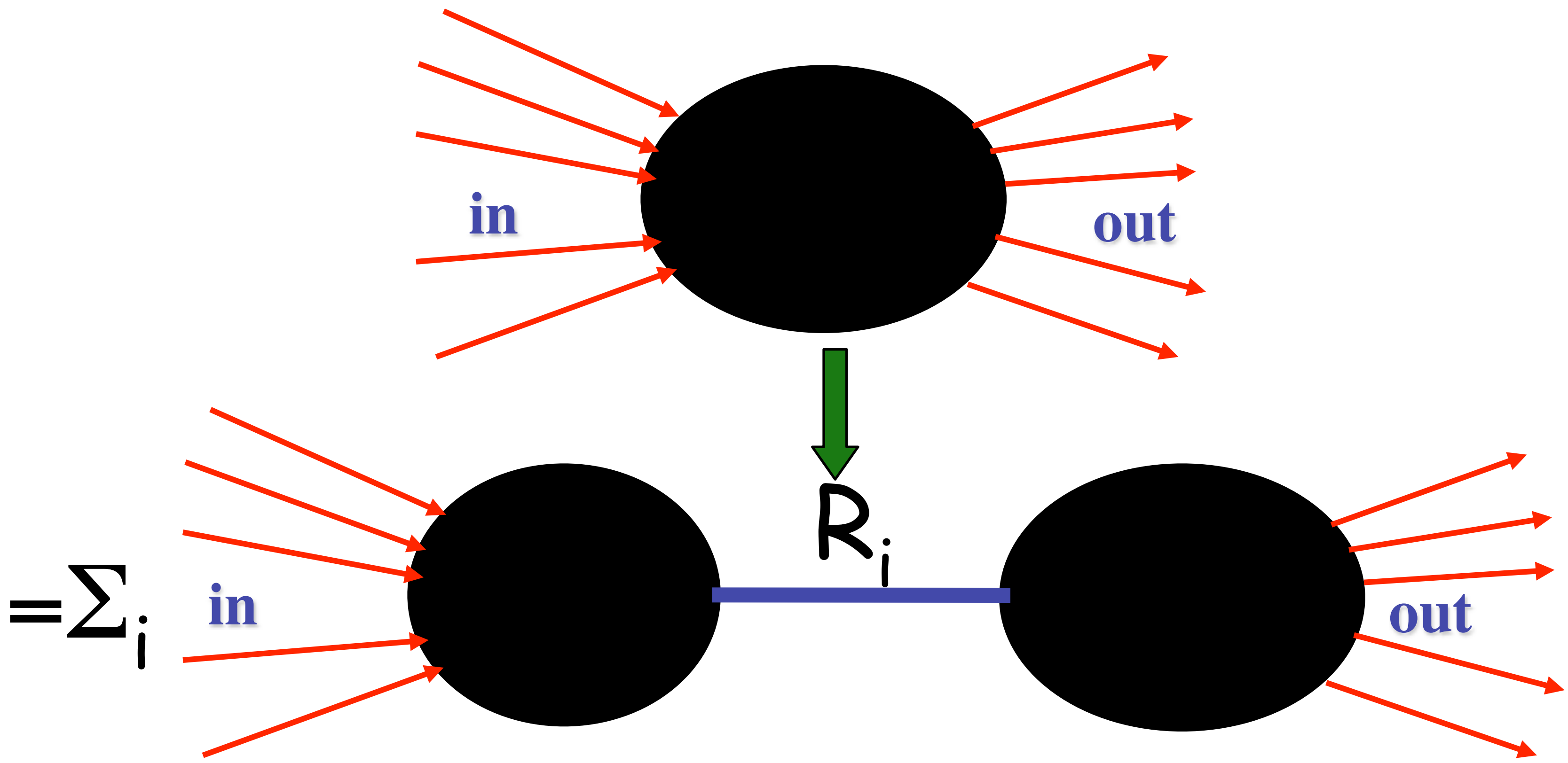
- The properties of the Beta-function were very nice and welcome, almost too good to be true.
- There was, however, a big worry based on previous experience: possibly, in order to satisfy all the constraints, the model had to contain "ghosts", i.e. negative-norm states produced with negative probability.
- If so the model would have been inconsistent.
- In order to find out, it was necessary to identify first all the states.

To fully disentangle the spectrum we need to construct more general scattering amplitudes and use a basic property of each single intermediate state known as factorization. Each state contributes to the residue of the corresponding pole by the product of its couplings to the “initial” and “final” states.

This is what unitarity of the S-matrix reduces to in the single-particle-exchange approximation.

Thus counting states amounts to answering the following question:

Q: How many terms are needed (in the sum over i) in order to have, for all **in** and **out** states,




We need generalization of B-function to n -> m processes

# Dual Resonance Models
# (Multiparticle generalizations of the B-function)

How should we generalize the duality properties of two-body scattering that allowed us to find the solution?
We will insist on having poles (and only poles) in the appropriate Mandelstam variables as well as the appropriate crossing symmetries.
(Multi)Regge behaviour will come out as a bonus.
The other crucial input will be imposing "Planar Duality".
***************

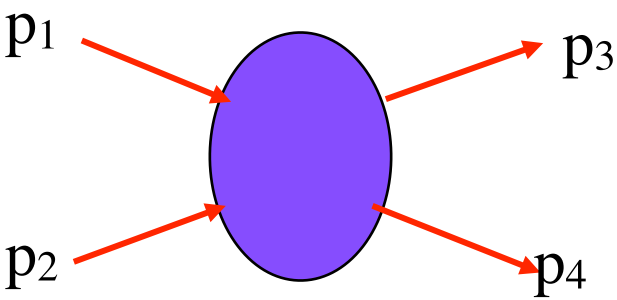
NB. There is also a notion of "Non-Planar-Duality", embodied in the Shapiro-Virasoro model, later interpreted as describing the interaction of closed strings.

# Planar Duality (to be related to open strings)

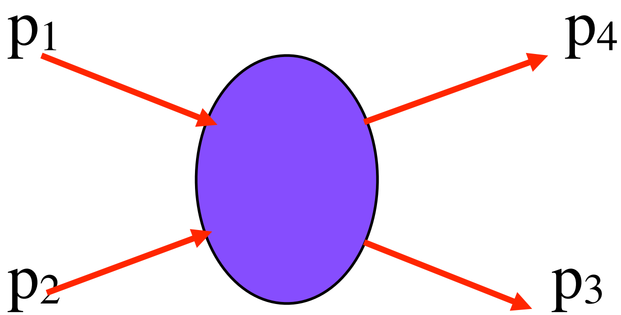
The Beta-function model for the 4-point function (2->2 scattering) exhibits "planar duality" i.e. duality w.r.t. the channels put in evidence by each particular order of the external lines. There are 3 of them (3 pairs of Mandelstam variables):

$$s = -(p_1 + p_2)^2 \quad , \quad t = -(p_1 - p_3)^2 \quad , \quad u = -(p_1 - p_4)^2$$

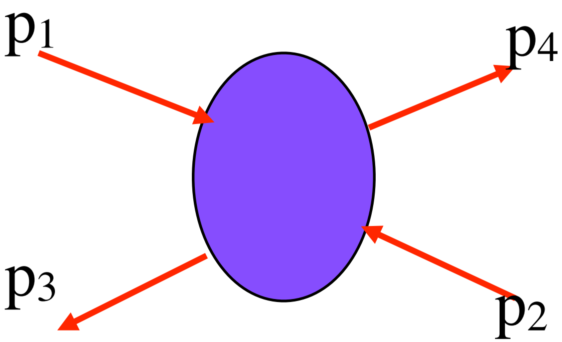
s-t duality



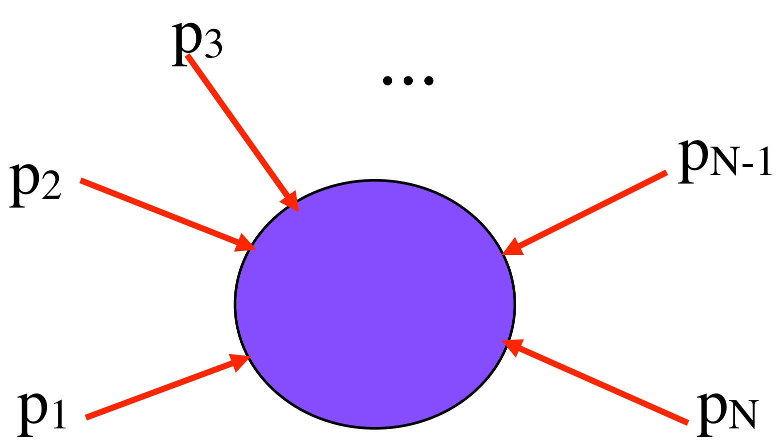

s-u duality




t-u duality



Consider now a process involving $N > 4$ external spinless particles. The corresponding (connected) amplitude is called an N-point function $A_N$. There are $(N-1)!/2$ distinct terms (distinct cyclic orderings) that have to be added at the end (with some specific numerical weights). Consider the term corresponding to the "trivial" cyclic ordering:



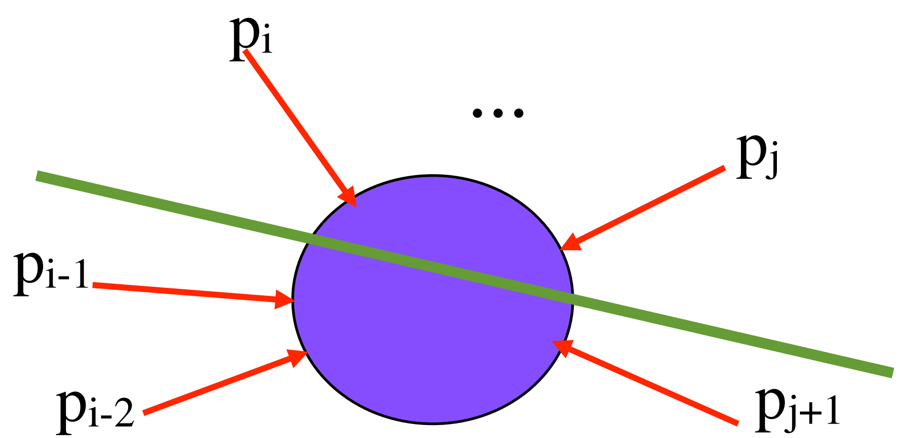

It will be given by an analytic function with poles in the Mandelstam invariants corresponding to its "planar channels".

Useful convention: all momenta are incoming so that 4-momentum conservation reads: $\Sigma_i \; p_i = 0$.

=> some of the $p_{i0} = E_i$ must be negative.

They correspond to outgoing particles w/ 4-momentum $-p_i$

planar channels are defined by a partition of the external legs in two sets of adjacent legs each containing at least two particles



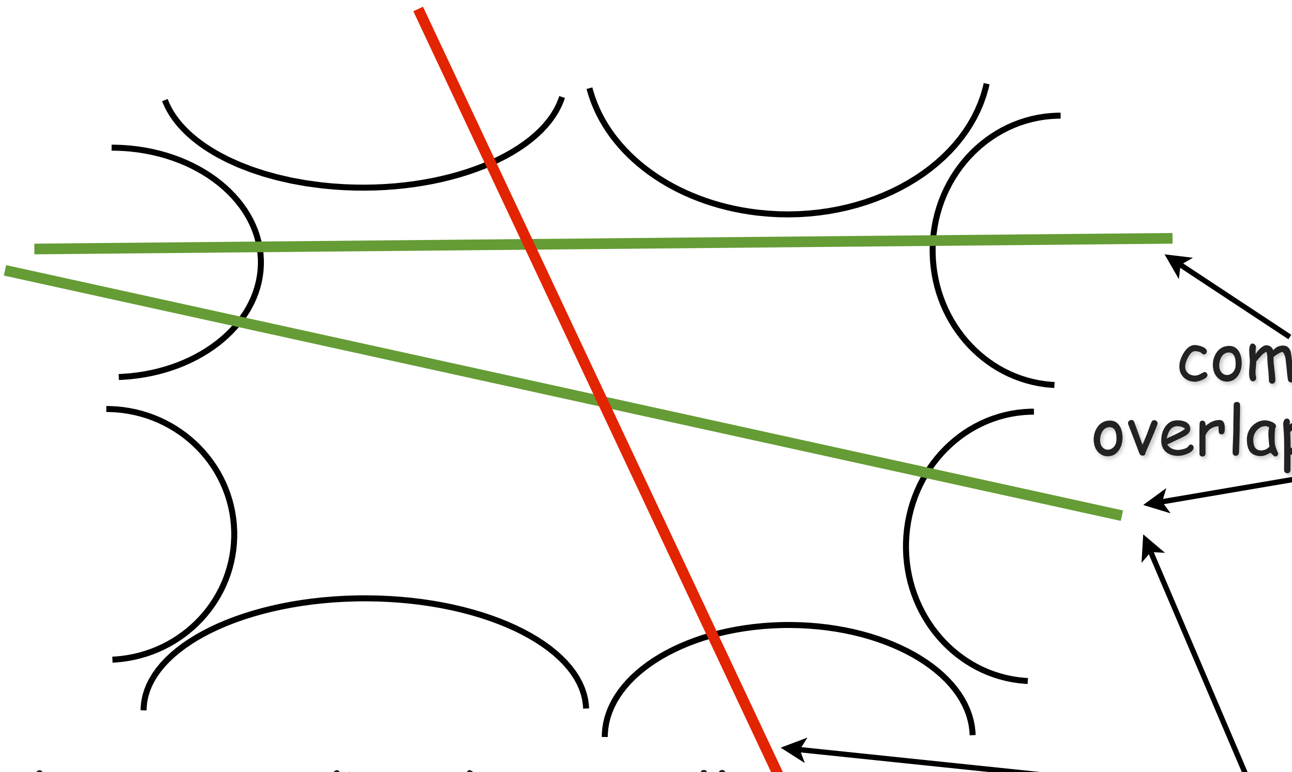

Poles appear in the corresponding Mandelstam variables:

$s_{ij} = -(p_i + p_{i+1} + \ldots + \; p_j)^2 = -(p_{j+1} + p_{j+2} + \ldots + \; p_{i-1})^2$

Their total number is $N(N-3)/2$ (= 2, 5, 9, ...)

# Illustration of planar duality

## (in)compatible = simultaneous poles (in)possible

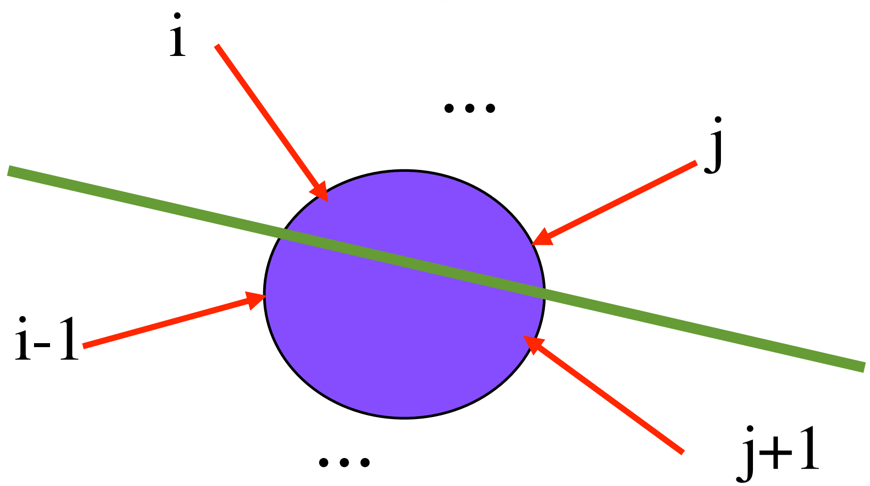


# Koba-Nielsen form for the N-point function

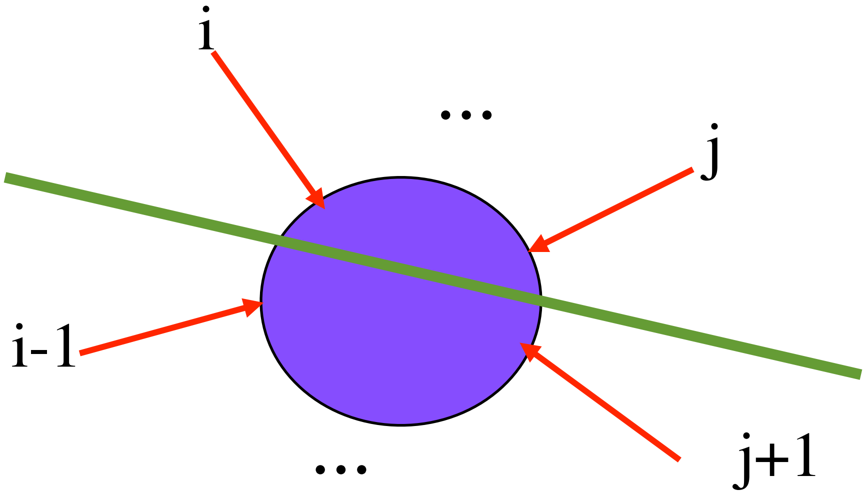
(for the special case of $\alpha(0) = 1$. Reason: see below)

The most useful form for the N-point function was given by Koba & Nielsen (1968). It has the advantage of treating all the external particles on the same footing. Their construction is as follows:

Associate with each external particle a real variable $z_i$ ($i = 1,2, \ldots N$) and to each planar channel a particular anharmonic ratio of the z's:




$$(z_i, z_{i-1}, z_j, z_{j+1}) \equiv \frac{(z_i - z_j)(z_{i-1} - z_{j+1})}{(z_{i-1} - z_j)(z_i - z_{j+1})}$$




$$(z_i, z_{i-1}, z_j, z_{j+1}) \equiv \frac{(z_i - z_j)(z_{i-1} - z_{j+1})}{(z_{i-1} - z_j)(z_i - z_{j+1})}$$

$B_N$ is then given by (a,b,c are chosen arbitrarily):

$$
\begin{aligned}
B_N &= \int_{-\infty}^{+\infty} dV(z) \prod_{i,j} (z_i, z_{i-1}, z_j, z_{j+1})^{-1-\alpha(s_{ij})} \\
dV(z) &= \frac{\prod dz_i \theta(z_i - z_{i+1})}{\prod (z_i - z_{i+2}) dV_{abc}} \\
dV_{abc} &= \frac{dz_a dz_b dz_c}{(z_b - z_a)(z_c - z_b)(z_a - z_c)}
\end{aligned}
$$

$$
\begin{aligned}
B_N &= \int_{-\infty}^{+\infty} dV(z) \prod_{i,j} (z_i, z_{i-1}, z_j, z_{j+1})^{-1-\alpha(s_{ij})} \\
dV(z) &= \frac{\prod dz_i \theta(z_i - z_{i+1})}{\prod (z_i - z_{i+2}) dV_{abc}} \\
dV_{abc} &= \frac{dz_a dz_b dz_c}{(z_b - z_a)(z_c - z_b)(z_a - z_c)}
\end{aligned}
$$

Integrand and integration measure are invariant under projective O(2,1) transformations:

$$
z_i \to \frac{\alpha z_i + \beta}{\gamma z_i + \delta} \; ; \; \alpha\delta - \beta\gamma = 1
$$

Without dividing by $dV_{abc}$ one would get infinity.

3 z's can be fixed arbitrarily leaving N-3 int. variables.

$$
\begin{aligned}
B_N &= \int_{-\infty}^{+\infty} dV(z) \prod_{i,j} (z_i, z_{i-1}, z_j, z_{j+1})^{-1-\alpha(s_{ij})} \\
dV(z) &= \frac{\prod dz_i \theta(z_i - z_{i+1})}{\prod (z_i - z_{i+2}) dV_{abc}} \\
dV_{abc} &= \frac{dz_a dz_b dz_c}{(z_b - z_a)(z_c - z_b)(z_a - z_c)}
\end{aligned}
$$

Using relations such as:

$$
\gamma_{ij} = \alpha(s_{ij}) + \alpha(s_{i+1,j-1}) - \alpha(s_{i+1,j}) - \alpha(s_{i,j-1}) = -2\alpha' p_i p_j
$$

we collect all the factors that contain a given ($z_i$-$z_j$) and obtain the standard KN form:

$$
B_N = \int_{-\infty}^{+\infty} \frac{\prod dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \prod_{j>i} (z_i - z_j)^{2\alpha' p_i \cdot p_j}
$$

$$B_N = \int_{-\infty}^{+\infty} \frac{\prod dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \prod_{j>i} (z_i - z_j)^{2\alpha' p_i \cdot p_j}$$

Note that the integrand is now independent of the cyclic ordering of the external lines. This only appears in the integration measure through the ordering of the z's.

A convenient choice for the 3 fixed z's is:

$$z_a = z_1 = \infty \ ; \ z_b = z_2 = 1 \ ; \ z_c = z_N = 0$$

$$B_N = \prod_{3}^{N-1} \left[ \int_0^1 dz_i \theta(z_i - z_{i+1}) \right] \prod_{i=2}^{N-1} \prod_{j=i+1}^{N} (z_i - z_j)^{2\alpha' p_i \cdot p_j}$$

$$B_N = \prod_3^{N-1} \left[ \int_0^1 dz_i \theta(z_i - z_{i+1}) \right] \prod_{i=2}^{N-1} \prod_{j=i+1}^{N} (z_i - z_j)^{2\alpha' p_i \cdot p_j}$$

This was the starting point of the original study of the spectrum (FV & BM, 1969).

By far the simplest way to describe it is by introducing (Fubini Gordon & GV, Nambu, 1969) an operator formalism (also leading straight into string theory!).

$$[q_\mu \ , \ p_\nu] = i\eta_{\mu\nu} \ , \quad [a_{n,\mu} \ , \ a^\dagger_{m,\nu}] = \delta_{n,m}\eta_{\mu\nu} \ , \quad \eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$$

$$(n = 1, 2, \ldots \ ; \ \mu = 0, 1, 2, \ldots D - 1)$$

# Proof of factorization

Two essential ingredients:

1) a (FV) "field operator" $Q_\mu(z)$ and

2) a (FV&G) "vertex operator" V(z, k)

$$Q_\mu(z) = Q_\mu^{(0)}(z) + Q_\mu^{(+)}(z) + Q_\mu^{(-)}(z) \quad ; \quad Q_\mu^{(0)}(z) = q_\mu - 2i\alpha' p_\mu log z$$

$$Q_\mu^{(+)}(z) = i\sqrt{2\alpha'} \sum_{n=1}^{\infty} \frac{a_{n,\mu}}{\sqrt{n}} z^{-n} \quad ; \quad Q_\mu^{(-)}(z) = -i\sqrt{2\alpha'} \sum_{n=1}^{\infty} \frac{a_{n,\mu}^\dagger}{\sqrt{n}} z^{n}$$

$$V(z,k) =: e^{ik\cdot Q(z)} : \ \equiv \ e^{ik\cdot Q^{(-)}(z)} \ e^{ik\cdot q} \ e^{2\alpha' k\cdot p log z} \ e^{ik\cdot Q^{(+)}(z)}$$

They satisfy the following operator identities:

$$[Q_\mu^{(+)}(z), Q_\nu^{(-)}(w)] = -2\alpha' log\left(1 - \frac{w}{z}\right)\eta_{\mu\nu}$$

$$V(z,k)V(w,k') =: V(z,k)V(w,k') : (z-w)^{2\alpha' k\cdot k'}$$

leading to:

$$\langle 0| \prod_{i=1}^{N} V(z_i, p_i)|0\rangle = (2\pi)^D \delta^{(D)}(\sum p_i) \prod_{i>j} (z_i - z_j)^{2\alpha' p_i\cdot p_j}$$

Recalling $B_N = \int_{-\infty}^{+\infty} \frac{\prod dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \prod_{j>i} (z_i - z_j)^{2\alpha' p_i\cdot p_j}$

we have the elegant result:

$$(2\pi)^D \delta^{(D)}(\sum p_i) B_N = \int_{-\infty}^{+\infty} \frac{\prod dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \langle 0| \prod_{i=1}^{N} V(z_i, p_i)|0\rangle$$

This looks already nicely factorized. To complete the proof we use the fact that the operator

$$L_0 = \alpha' p^2 + \sum_{n,\mu} n \; a^\dagger_{n,\mu} \; a^\mu_n$$ acts on Q as z d/dz Q giving:

$$V(z,k) = z^{L_0 - \alpha' k^2} V(1,k) z^{-L_0} = z^{L_0 - 1} V(1,k) z^{-L_0}$$

Using this repeatedly, performing the integrals on $z_{i+1}/z_i$ we arrive at the desired fully factorized form:

$$\begin{aligned}(2\pi)^D \delta^{(D)}(\sum p_i) B_N &= \langle p_1 | V(1,p_2) \; D \; V(1,p_3) \; D \; V(1,p_4) \; D \ldots D \; V(1,p_{N-1}) | p_N \rangle \\ D &= \frac{1}{L_0 - 1}\end{aligned}$$

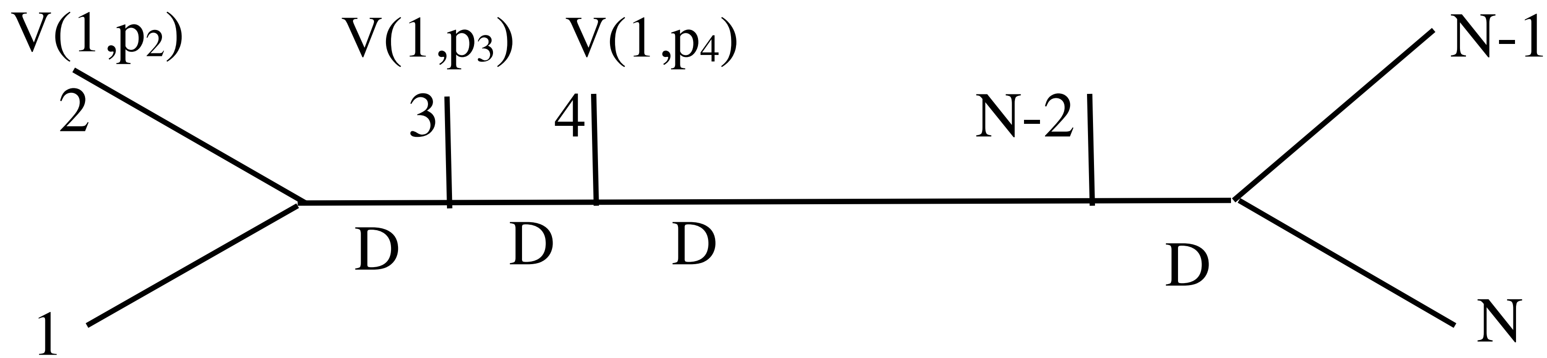
Introducing a complete set of harmonic oscillator states before and after a given "propagator" D will provide a pole at:

$$L_0 = 1 \Rightarrow -\alpha' p^2 = \alpha' M^2 = -1 + \sum_{n,\mu} n \; a^\dagger_{n,\mu} \; a^\mu_n = -1 + \sum_{n,\mu} n N_{n,\mu}$$

$$(2\pi)^D \delta^{(D)}(\sum p_i) B_N = \langle p_1 | V(1,p_2)\ D\ V(1,p_3)\ D\ V(1,p_4)\ D \ldots D\ V(1,p_{N-1}) | p_N \rangle$$

$$D = \frac{1}{L_0 - 1}$$

$$1 = \sum \int dk | N_{n,\mu}, k \rangle \langle N_{n,\mu}, k |$$




$$L_0 = 1 \Rightarrow -\alpha' p^2 = \alpha' M^2 = -1 + \sum_{n,\mu} n\ a^\dagger_{n,\mu}\ a^\mu_n = -1 + \sum_{n,\mu} n N_{n,\mu}$$

We conclude that a sufficient set of states for factorization consists of the eigenstates of momentum and of the occupation numbers of our harmonic oscillators i.e.

$$|N_{n,\mu}, k\rangle \sim \prod_{n,\mu} \left(a^{\dagger}_{n,\mu}\right)^{N_{n,\mu}} e^{iqk}|0\rangle \quad ; \quad a_{n,\mu}|0\rangle = p_{\mu}|0\rangle = 0$$

$$-\alpha' k^2 = \alpha' M^2 = -1 + \sum_{n,\mu} n \; a^{\dagger}_{n,\mu} \; a^{\mu}_{n}$$

Because of the "wrong" sign of the timelike c.r., states created by an odd number of timelike operators are ghosts. Was the DRM doomed? One (tiny?) hope remained: all those states were sufficient but perhaps only a (ghost-free?) subset was necessary.Decoupling of the lightest ghost was already noticed by F&V...

The “ghost hunting” project was a “tour de force” that culminated in the proof of a “no-ghost theorem” by Brower and by Goddard & Thorn.

At the basis of the theorem was the discovery of the Virasoro operators (needed to construct the spurious/ physical states) and of their algebra, and the explicit construction of an infinite set of positive-norm physical (DDF) states (using only D-2 components of the operators)

There were a couple of prices to pay for the absence of ghosts: the Regge intercept, $\alpha_0$, had to be exactly 1 (implying a massless spin one particle and a spin zero tachyon) and the dimensionality of spacetime had to be less than (or equal to) 26.

At exactly D=26 the physical Hilbert space would be completely spanned by the DDF states corresponding to oscillators in (D-2)=24 dimensions.

Meanwhile, C. Lovelace had shown that loops were consistent with unitarity only if D=26.

For D=26 and $\alpha_0$=1 the model looked consistent except for the tachyon ($M^2 = -1/\alpha'$).

It took a while before it was realized that the DRM was a theory of strings. Till about 1972 it looked like a very strange kind of theory, mysteriously different from anything that had been seen before, like QFT or GR.

As such it polarized the community with the opponents (particularly within the establishment) outnumbering the (mostly young) enthusiasts.

Even modern string theory remembers its very unconventional origins and uses concepts and methods that are very different from those people are accustomed to

"A piece of physics from the 21st century that fell, by accident, in the 20th"

(S. Fubini, ~1969)