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De la corde hadronique à la gravitation quantique... et retour

Cours III : 5 mars

Cordes et gravitation quantique

Université Claude Bernard Lyon 1

UNIVERSITÉ DE LYON

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Classical vs. Quantum strings

- We have already seen that there are non-trivial constraints on the dimensionality of space-time in which a quantized string can consistently propagate.
- We have also seen that the spectrum of quantum strings is strongly constrained ($\alpha_0 = 1$).
- These are just examples of many properties that distinguish classical from quantum strings.
- For the moment, let's focus on two crucial ones.

Emergence of a fundamental length

- Classical string theory is scale-invariant in Minkowski spacetime. It has no characteristic length (Cf. Classical GR).
- Given a solution of the equations of motions and constraints we can generate another solution by multiplying all the string coordinates by the same arbitrary factor k.
- The new solution is a string whose M(J) is k times (k^2 times) larger. J/M^2 remains the same.
- We can also trivially change T (S_{NG} is rescaled)

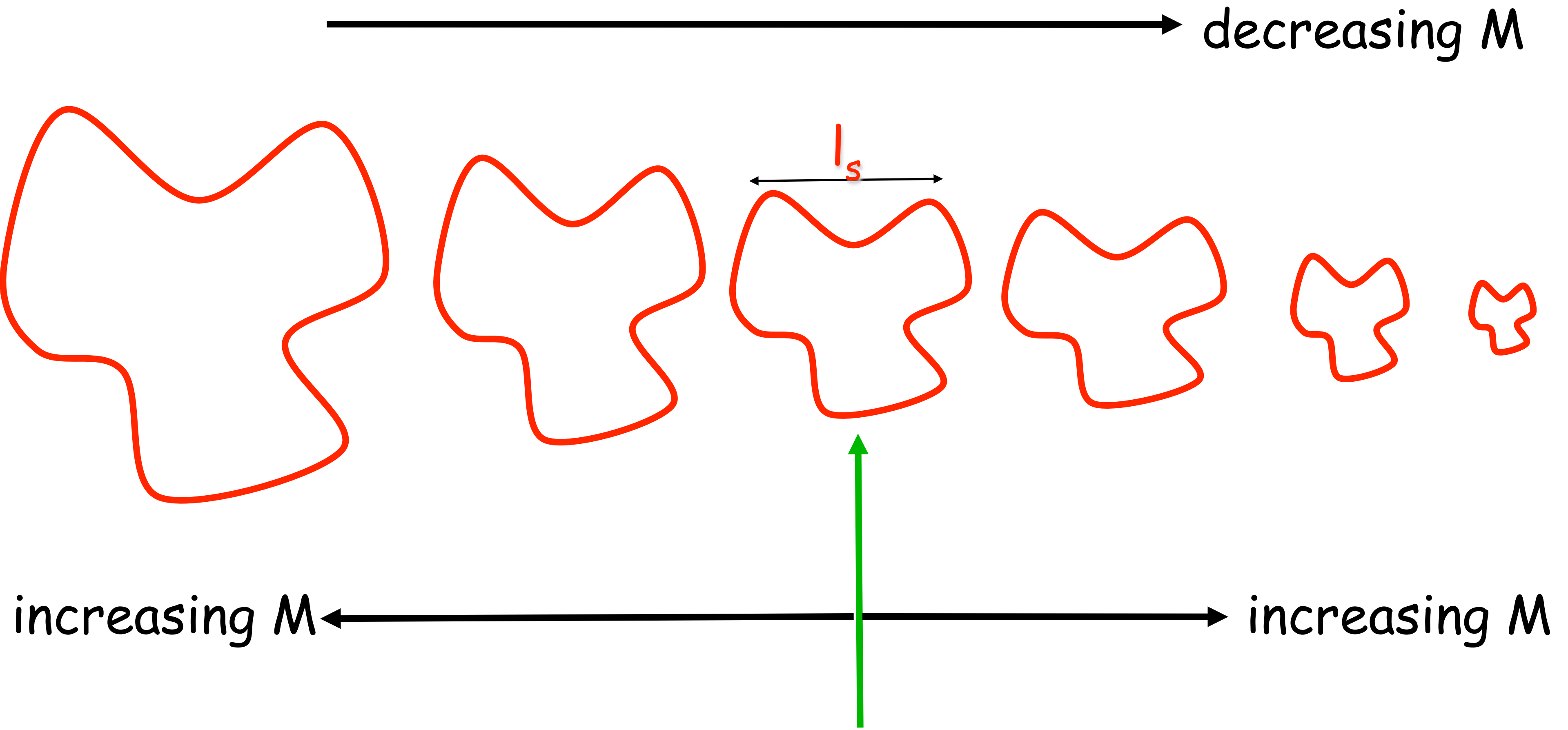
- In the quantum theory the relevant quantity is the dimensionless ratio S/h. Since:

$$\frac{1}{\hbar} S_{NG} = -\frac{T}{\hbar}(\text{Area swept}) \equiv -\frac{1}{\pi l_s^2}(\text{Area swept}) \quad ; \quad l_s^2 \equiv 2\alpha'\hbar$$

quantization introduces a fundamental length, l_s. The ratio of a generic string's size and l_s is now a relevant dimensionless parameter (even in Minkowski: in curved spacetime there are further relevant ratios...).

- This fundamental length enters string theory in many ways. It is the characteristic size of a (minimal-mass) string (Cf. ground st. of h.o.).
- NB: In the string literature l_s^2 and $2\alpha'$ are often identified. This sometimes leads to some puzzles...

Without QM strings become lighter and lighter as they shrink



With QM strings are lightest when their size is l_s

Angular momentum without mass

A classical string cannot have angular momentum without having a finite length, hence a finite mass. The rigid-rod solution maximizes J/M^2. A quantum string, instead, can have up to 2 units of J without gaining mass. The fact that this is a quantum effect is clear:

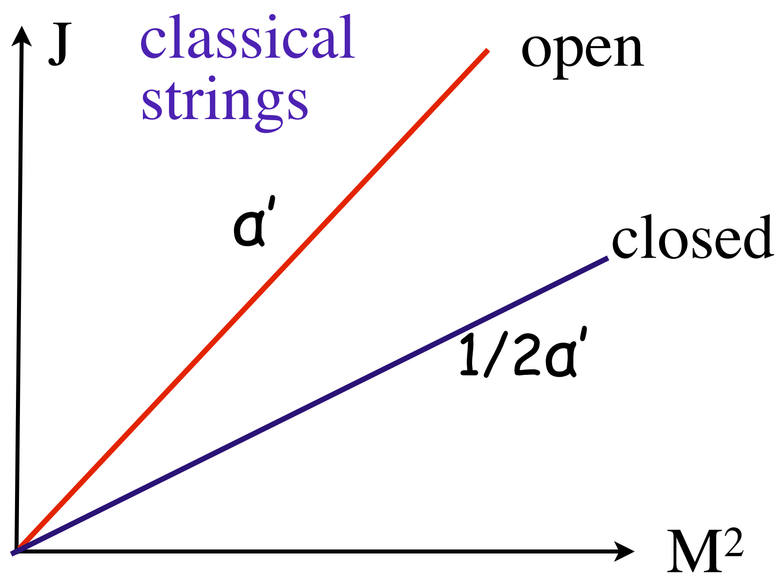
after consistent regularization

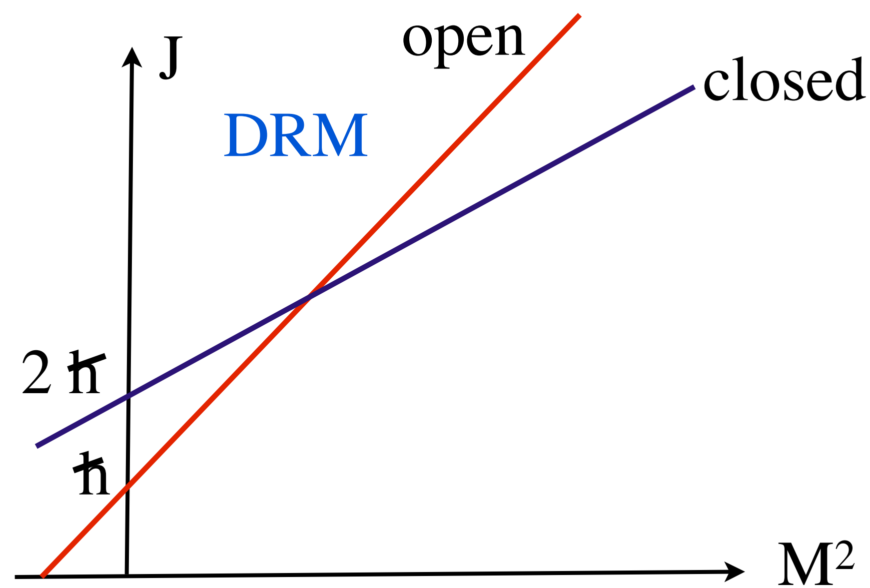
$$\frac{M^2}{2\pi T} \geq J + \hbar \sum_{1}^{\infty} \frac{n}{2} = J - \alpha_0 \hbar \qquad \alpha_0 = 0, \frac{1}{2}, 1, \frac{3}{2}, 2.$$

α_0 =1 and 2 correspond to open and closed bosonic strings, respectively. In the superstring there are also massless J = 1/2, 3/2 states.

The classical limit corresponds to taking strings that are large in l_s units. They correspond to large occupation numbers i.e. to heavy strings. This is where the two graphs shown below agree with each other.

On the contrary, for small, light strings the classical picture fails. Only quantum strings can be interesting for a unified description of fundamental forces and particles!





Shortcomings of the bosonic string

1. Presence of a tachyon
2. Absence of fermions
3. $D \neq 4$

Presence of a Tachyon

Tachyons are a problem but not necessarily a killer. In QFT a tachyon is a sign of instability of the (false) vacuum state around which we carry out quantization and perturbation theory.

The Higgs model for the SSB of SU(2)xU(1) is the most famous example of how we can use a tachyon to our own advantage.

Unfortunately it was (and to a large extent still is) not at all clear how to change the vacuum in String Theory.

This is why people tried to find tachyon-free models with a nice perturbative expansion.

Absence of fermions

This was a very obvious shortcoming of the bosonic DRM. After all one wanted to describe also protons and neutrons besides mesons!

$$D \neq 4$$

This was another big disappointment. People tried to find out whether by adding further degrees of freedom one could get down to D=4. They only succeeded half way...

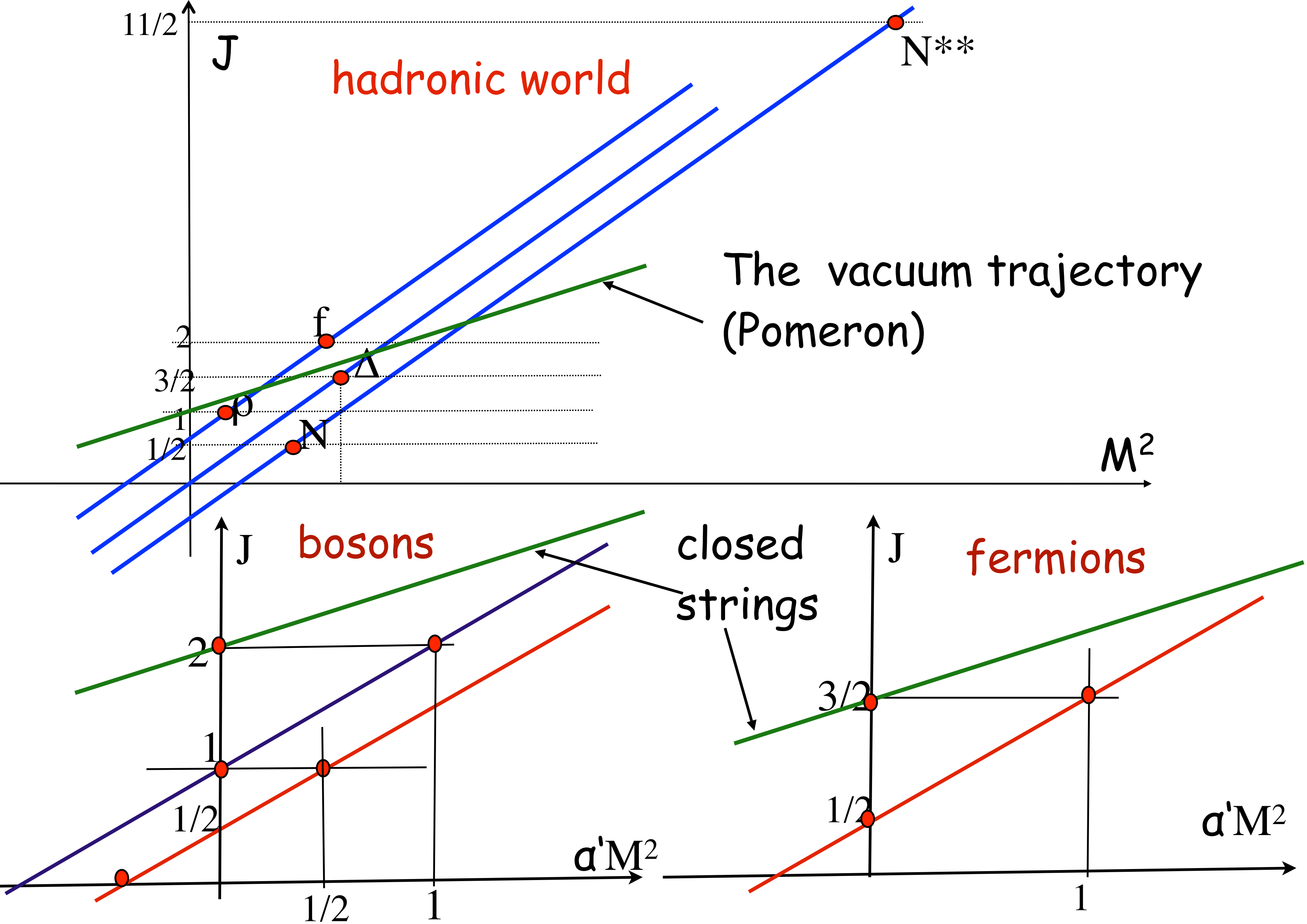
Adding world-sheet fermions

Even before the string reinterpretation of the DRM Neveu & Schwarz and Ramond were able to generalize the operator formalism by adding to the bosonic field $Q(z)$ a Grassmann (i.e. anticommuting) field $\psi(z)$.

The action for the bosonic string can be generalized by adding to the string coordinate $X^\mu(\xi)$ a fermionic "coordinate" $\psi^\mu{}_\alpha(\xi)$ which is a two-component spinor in 2-dimensions (a world-sheet spinor) but, like X^μ, is a spacetime vector (in D dimensions).

The model becomes consistent in 10 spacetime dimensions and with some specific (integer and half integer) intercepts.

It has two-dimensional (world-sheet) supersymmetry but not a spacetime (10-dimensional) SUSY as seen in the low-lying spectrum.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The vacuum trajectory
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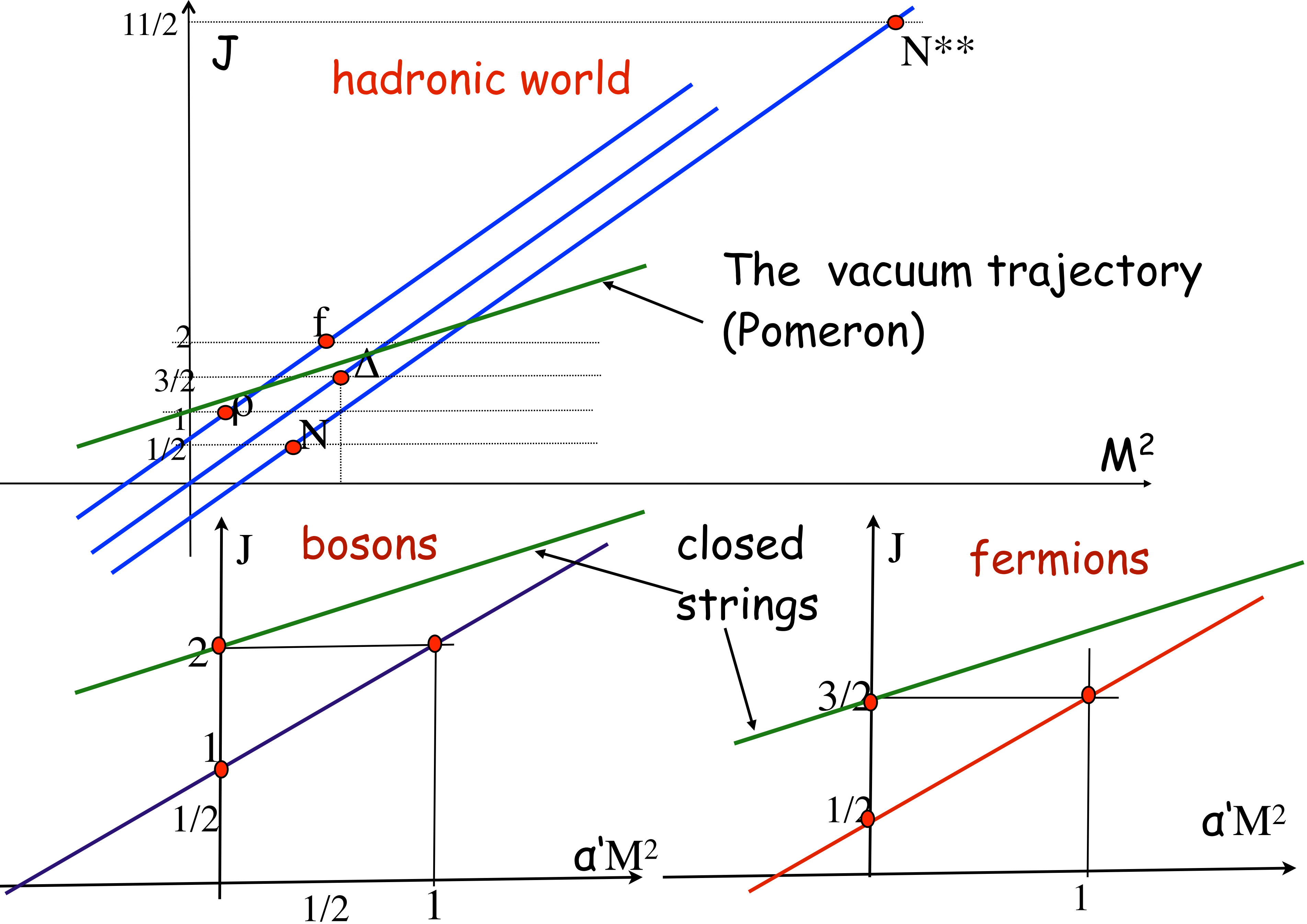
GSO projection and the Superstring

The NSR model still has a tachyon. In 1976, Gliozzi, Scherk and Olive found a smart way to eliminate it. They introduced a WS fermion "parity" P_F.

The tachyon has $P_F=-1$, the massless vector $P_F=+1$

GSO then proved that P_F is conserved in the NSR model so that a projection on external states with $P_F=+1$ is consistent with factorization ($P_F=-1$ states do not appear as intermediate states). The tachyon is eliminated. Also, half of the fermions in the fermionic sectors are projected out.

The fermionic ground state becomes a Majorana-Weyl spinor (MW spinors only exist in D = 4n+2) in D=10. It has 8 ($2^5/4$)components (just like a massless vector which has 10-2 = 8) and is chiral (in the 10-dimensional sense) which makes it potentially interesting for phenomenology.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A counting of states shows that, not only the massless spectrum, but also the excited states contain the same number of bosons and fermions.

The (spacetime) supersymmetry of the spectrum can be generalized to interactions and is a true symmetry of string theory after the GSO projection.

For closed strings one applies GSO separately to left- and right-movers. However, we can use either the same or an opposite GSO projection for left and right-movers. In the former case we have a chiral theory, called Type IIB, in the latter a non-chiral theory, called Type IIA.

The open superstring theory can have an associated gauge group (via Chan-Paton factors) and chiral fermions, and is called Type I.

Phenomenological shortcomings

With its string reinterpretation the DRM had become, around 1972-73, a respectable theory.

The absence of ghosts had been a remarkable achievement, like that of adding consistently fermions.

Some qualitative features of the model were in striking agreement with experiments, in particular the linearity of the Regge trajectories, their universal slope, and the degeneracy of even and odd-signature trajectories implied by DHS duality.

Other features, however, were in striking disagreement with the data:

1. $D \neq 4$;

2. Presence of massless particles (and of tachyons before GSO). More generally, the low-lying states were not what one wanted for hadrons.

However, until then, one could nourish some hope that, by working harder, those problems could be overcome:

1. One had already been able to reduce D from 26 to 10.

Why not to 4? (adding more SUSY on the WS brings down to D=2!);

2. We knew how to deal with tachyons in QFT and how to give mass to massless gauge bosons via the Higgs mechanism. Why not try to do the same in string theory?

The real killer was softness!

String theory is "soft" i.e. does not allow "hard" processes in which two colliding strings exchange a large momentum. Such processes are exponentially damped at high E.

Experimentally, there was mounting evidence that "hard" processes are not so rare in hadronic physics:

1. $R = \sigma(e^+ e^- \to \text{hadrons})/\sigma(e^+ e^- \to \mu^+ \mu^-) \to$ constant.
2. Bj scaling in $e^- p \to e^- + X$ (SLAC) => partons?
3. Large p_t events in pp scattering at the ISR (CERN).
4. Form factors at large q^2.

All evidence for point-like structure in the hadrons.

Even worse was "competition"

~ 1973 QCD came about with its

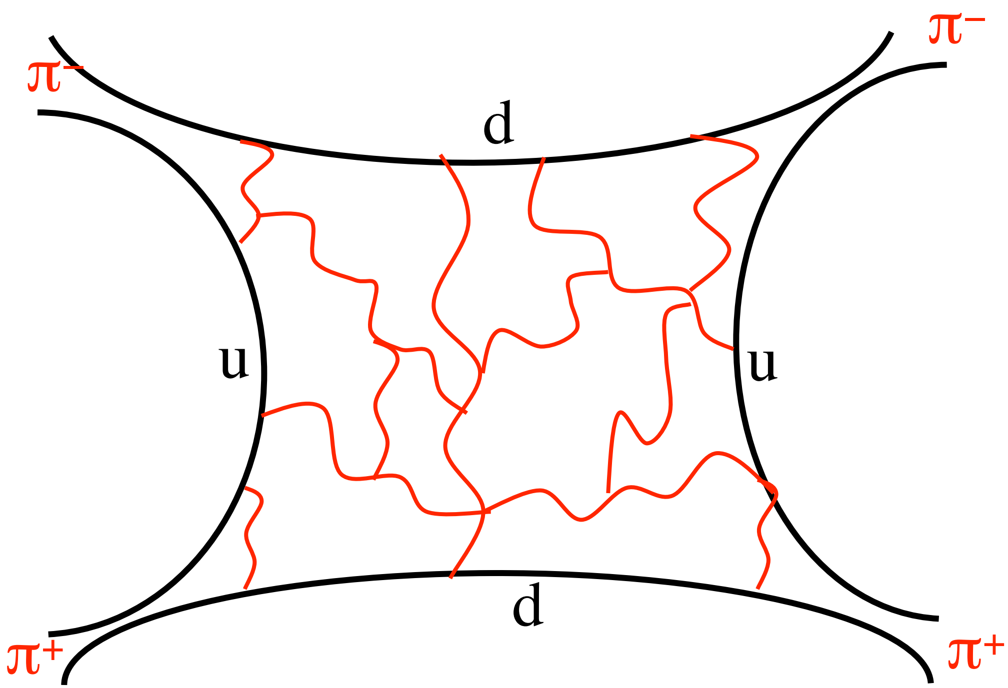
1. Ultraviolet (asymptotic) freedom that could explain hard processes from the existence of point-like structures (quarks and gluons) inside the hadrons.
2. Conjectured infrared slavery (confinement) explaining why we do not see free quarks and gluons.
3. Furthermore, quark confinement would be realized through the formation of a narrow chromo-electric flux tube (dual Meissner effect) simulating a string stretched between a quark and an antiquark...

Yet it was (psychologically?) difficult to give up: What about DHS duality and the topological structure of string theory's perturbation theory, so much unlike that of any "normal" QFT?

I gave up (~1974), when 't Hooft showed that even topology comes out of QCD, provided one considers a 1/N expansion....

In SU(N) QCD, at large N, duality diagrams acquire a precise meaning: they are planar Feynman diagrams bounded by quark propagators & filled with gluons.

NB: this is not usual perturbation theory and has DHS duality



- These planar diagrams give, at leading order, the zero-width approximation we had been using all the time.
- At next-to-leading order the non-planar diagrams should give new quarkless bound states, the glueballs, and presumably the Pomeron as the Regge trajectory glueballs lie on (corresponding to closed strings).
- The Hagedorn temperature is re-interpreted as a deconfining temperature for quarks and gluons.
- It all seemed to fall nicely into place…
- Was that beautiful theoretical construction completely worthless?
- Hard to believe but, for about 10 years (1974-1984), most theorists stopped working on strings.

The zero-slope limit

(Scherk '71, Neveu-Scherk '72, Yoneya '72-'73)

We have argued that Quantum String Theory (QST) reduces to Classical String Theory (CST) when the size L of the string is large compared the fundamental length of QST:

$$l_s^2 \equiv 2\alpha' \hbar$$

Obviously, the classical limit corresponds to h --> 0 but one could think that it also corresponds to the limit α'--> 0.

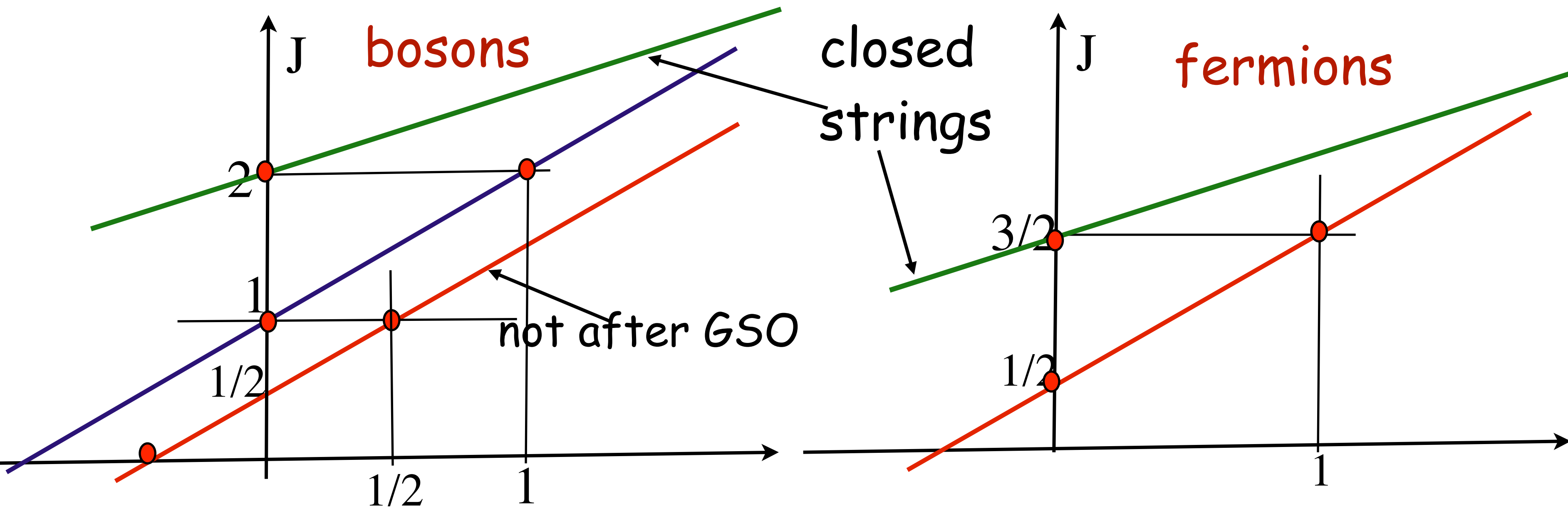
This latter statement is wrong. Indeed:

$$L = \alpha' M >> l_s = \sqrt{\alpha' \hbar} \Rightarrow M \gg \sqrt{\frac{\hbar}{\alpha'}} \equiv M_s$$

- $M_s = h/l_s$ is the characteristic mass/energy scale of string theory providing its typical excitation energy/level spacing.
- The CST limit is rather α'--> infinity.

Q: What is then the α'--> 0 limit? The answer is that QST goes over to conventional QFT! QST is thus an extension of QFT, very much like Relativity and QM are extensions of classical mechanics and reduce to it in the appropriate limits.

In the α'--> 0 limit the excited states are pushed to infinite mass and one is left with just the lowest states (which, as we have argued, are very quantum!). We are left with just the massless states...



The α'--> 0 limit can also be seen as the low-energy limit ($s \ll M_s^2$) for the interaction of massless strings.

What Neveu-Scherk (for open strings) and Yoneya (for closed strings) found is hardly surprising. At leading order in the momenta, the massless J=1 open string states couple like (abelian or non abelian) *gauge bosons*, while the massless J=2 closed string states couple exactly like a *graviton* in (semiclassical) *general relativity*.

This is what we should expect. *Gauge invariance* is needed for describing a massless J=1 particle, while *general covariance* is needed for describing a massless J=2 particle.

The successes of the Standard Model of elementary particles (the SM) and of *gravity* (GR) tell us that Nature likes such J= 1, 2 massless particles.

The Scherk-Schwarz proposal (1974)

- By 1974 nobody believed any more that QST could be the correct theory of strong interactions.
- Instead, the realization that at low-energy QST could reproduce gauge and gravitational interactions prompted Scherk and Schwarz to make a very bold conjecture.
- Could QST be used instead to describe the elementary particles of QCD, i.e. the quarks and the gluons themselves and then, why not, the gauge bosons of the other SM interactions and then, why not, the graviton and gravitational interactions? In short a TOE...
- Of course, a change in α' (l_s) was also necessary.

- A very bold proposal that had the advantage of turning (at least some) defeats into victories.
- The appearance of massless particles, a big embarrassment for strong interactions, was now a very welcome feature. QST predicted their existence, hence that of gauge and gravitational interactions.
- The softness of QST could solve the long standing problems with quantizing GR. Not only, it could even completely eliminate the UV problems of QFT.
- QST cried for SUSY, a possible solution to the hierarchy problem of the SM, if conveniently broken.

- The problem of D=10 still remained but people knew, since the work of Kaluza and Klein in the 20's, that the extra dimensions (6 in our case) could be made compact, become invisible as such, and provide a new mechanism for generating gauge interactions.
- In fact, with its fundamental length, QST could possibly solve a long-standing problem in KK theory: what fixes the size of the compact dimensions?

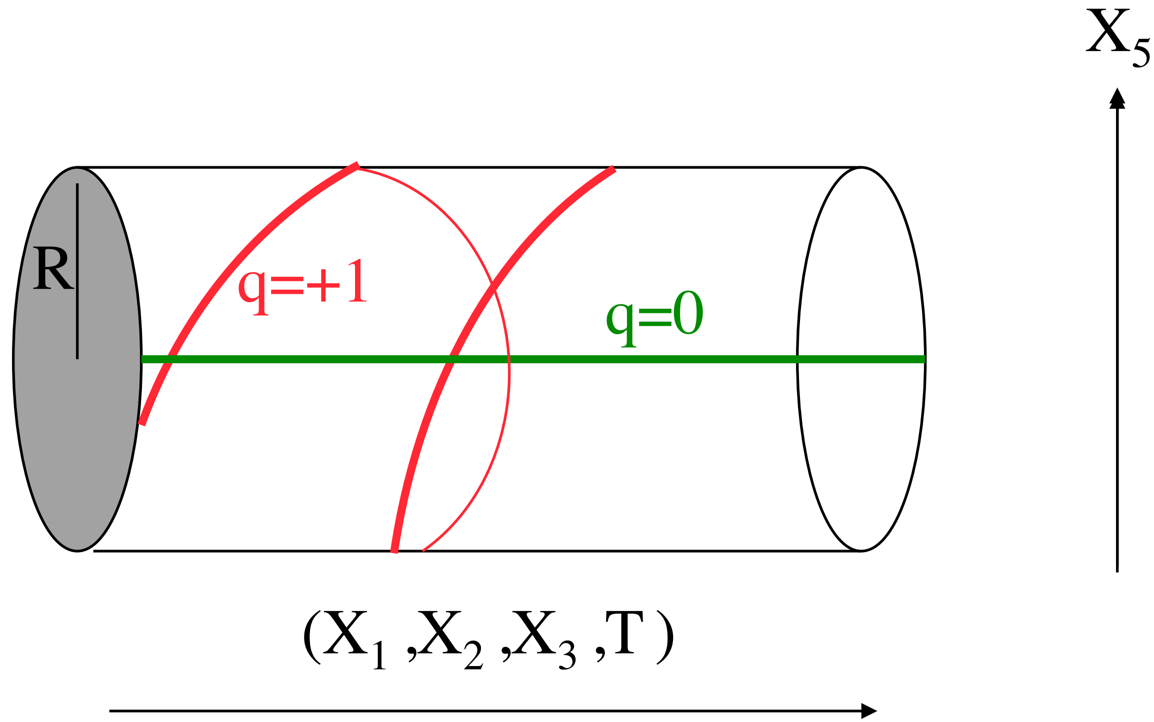
A quick reminder of KK theory

Kaluza (1921) and Klein (1926) (KK) managed to reformulate electromagnetism + gravity as just GR in a space containing one extra spatial dimension.

In KK theory the extra dimension of space is a circle of radius R.

The e.m. potential A_μ becomes the component $g_{\mu 5}$ of the 5-dimensional metric, while g_{55} is a scalar field associated with the (proper) radius of the circle.

After some initial skepticism, Einstein admitted that the KK idea was very appealing.



p_5 is quantized in units of h/R (QM is crucial)

$q = p_5/M_P = n\, l_P/R$, $n = 0, \pm 1, \pm 2, ..$

Quantization of electric charge automatic!

KK Unification $F_C = F_N$ at $E \approx hc/R = M_C\, c^2$.

M_C = mass of typical KK excitations.

QM is central to the KK idea.

The basic unit of electric charge (in natural units) becomes l_P/R, where $l_P \sim 10^{-33}$ cm is Planck's length, the fundamental length that can be constructed out of c, h and G_N.
Given $\alpha \sim 10^{-2}$, R should be ~ 10 l_P. But what fixes R itself? Why should it not shrink down to zero?
Another problem with conventional KK theory is that even QED becomes non-renormalizable in D=5!
These questions, left unanswered in KK theory, have interesting reformulations (and even answers) in QST (see next lecture)

- Not many people took the Scherk-Schwarz proposal seriously.
- The SM had just been completed with its strong and electroweak sectors. There was a lot to do, theoretically and experimentally, in order to work out predictions and to check them against the data. It was a very intense and fruitful period in QFT:
- large-N expansions, SUSY, computing hard processes in QCD, lattice gauge theories, instantons, U(1)&CP problems, FCNC, CP violation, GUTs...
- People could not care less about strings and Q-Gravity...
- Last but not least: it did not look easy to get chiral fermions from superstring theory.

The Green-Schwarz 1984 breakthrough

- Remember: The SM has chiral fermions but the matter content is such that all gauge anomalies cancel.
- It looked that, instead, string theory either had anomalies (of the QFT type) or could not have chiral fermions.
- Until 1984 it "almost" looked like a no-go theorem.
- But, in the summer of 1984, Green and Schwarz found the exception to the rule: TypeI superstrings allowed for a new kind of anomaly cancellation iff the gauge group was just SO(32)!
- This "1st revolution" relaunched string theory!