Fédération de recherche André Marie Ampère

Cours du Collège de France 2011-'12

Gabriele Veneziano

De la corde hadronique à la gravitation quantique... et retour

Cours IV : 12 mars

Cordes, Branes et Théories de Jauge

Université Claude Bernard Lyon 1
Université de Lyon
Ecole Normale Supérieure de Lyon

In my previous lectures I tried to explain how string theory arose from the phenomenology of strong interactions and, in particular, from the phenomenon of colour confinement, believed to be a property of QCD at large distances.

That string theory, however, could neither explain the low-lying hadronic spectrum nor the short-distance phenomena predicted by QCD's asymptotic freedom.

Instead, the properties of fundamental quantum strings looked well suited for describing elementary particles and fundamental (gauge and gravitational) interactions at a deeper (much shorter distance) level.

This motivated Scherk & Schwarz's 1974 proposal, but their suggestion was not taken seriously until 10 years later, i.e. until after the Green-Schwarz breakthrough.

In this last 3rd of the course we shall present some recent ideas according to which certain objects present in string theory allow to reformulate gauge theories (including the Higgs phenomenon) in a very appealing, geometrical language.

Finally, we will discuss a deep connection between gravitational quantities described by string theory in a certain background and corresponding quantities of a pure gauge theory (w/out gravity), the latter being defined on the boundary of that space-time (holography).

This gauge-gravity duality has the advantage of mapping a hard strong-coupling problem in gauge theory to an easy low-curvature problem on the gravity side.

Before getting there we have to introduce a few more important concepts.

Polyakov formulation of the bosonic string

Polyakov, building on previous work, has given an alternative formulation of string theory with the action:

$$S_P = -\frac{T}{2}\int d^2\xi\sqrt{-\gamma}\gamma^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu G_{\mu\nu}(X)$$

After expressing $\gamma_{\alpha\beta}$ in terms of X^μ and $G_{\mu\nu}$ using its eom one gets back the NG action (where $\gamma_{\alpha\beta}$ was NOT an independent field):

$$\gamma_{\alpha\beta} \equiv \frac{\partial X^\mu(\xi)}{\partial\xi^\alpha}\frac{\partial X^\nu(\xi)}{\partial\xi^\beta}G_{\mu\nu}(X(\xi))\ , \quad \alpha,\beta = 0,1 \quad , \quad \xi^0 = \tau,\ \xi^1 = \sigma$$

$$S_{NG} = -T\int d(Area) = -T\int d^2\xi\sqrt{-det\gamma_{\alpha\beta}} \equiv -T\int d\xi^0\int_0^\pi d\xi^1\sqrt{-det\gamma_{\alpha\beta}}$$

NB: Something similar can be done for point particles:

$$\tilde{S}_p = -\frac{1}{2}\int d\tau\left(e^{-1}\partial_\tau X^\mu\partial_\tau X^\nu g_{\mu\nu}(X) - e\ m^2\right)$$

Looks like a GR in 2 dimensions with X^μ as scalar fields.

World-sheet-reparametrizations become GCT in 2-d.

It is essential, for a string theory interpretation of Polyakov's action, that its local 2-dimensional symmetries are preserved at the quantum level (no anomalies). These are the above-mentioned GCT's and Weyl-invariance under a local rescaling of the whole metric (which turns out to be the difficult one!)

$$\gamma_{\alpha\beta} \to e^{2\lambda(\sigma,\tau)}\gamma_{\alpha\beta}$$

We shall be interested in considering more general "background fields" than just the metric $G_{\mu\nu}$.

Which other backgrounds can we add? Let us see how that works for point-particles.

A charged point-particle couples naturally to a vector potential without even invoking a 1D-metric:

$$S_A^{point} = q\int d\tau\ \dot{x}^\mu(\tau)\ A_\mu(x(\tau)) = q\int dx^\mu(\tau)\ A_\mu(x(\tau))$$

This action is invariant under the gauge transformation A--> A + dΛ .

In perfect analogy, a string naturally couples to a 2-form $B_{\mu\nu} = -B_{\nu\mu}$ without invoking a 2D-metric:

$$S_B = -\frac{T}{2}\int d^2\xi \epsilon^{\alpha\beta}\partial_\alpha X^\mu(\xi)\partial_\beta X^\nu(\xi) B_{\mu\nu}(X(\xi))$$

with $\varepsilon^{\alpha\beta}$ the Levi-Civita symbol in D=2. This action is invariant under the "B-gauge" transformation: $B_{\mu\nu}$ --> $B_{\mu\nu} + d_\mu\Lambda_\nu - d_\nu\Lambda_\mu$.

This can be easily generalized to higher-dimensional extended objects (p-branes)...

Can we write anything else that satisfies classically the 2D local symmetries, and in particular Weyl invariance? The only possibility appears to be:

$$S_\Phi = \frac{1}{4\pi}\int d^2\xi\sqrt{-\gamma}R(\gamma)\Phi(X(\xi))$$

but only if the field $\Phi(x)$, called the dilaton, is a constant.

In that case, the integral is proportional to the Einstein-Hilbert action, which, in D=2, has a topological meaning.

A well-known theorem states that :

$$\frac{1}{4\pi}\int d^2\xi\sqrt{-\gamma}R(\gamma) = 2(1-g)$$

where g is the genus of the Riemann surface (g=0 for the sphere, g =1 for the torus, etc.) described by the metric $\gamma_{\alpha\beta}$. Thus, if Φ is constant, $S_\Phi = 2\Phi(1-g)$. Let's discuss this case first, opening a short parenthesis on loops in QFT & QST.

In Feynman's path-integral approach to quantization we are supposed to integrate over the dynamical variables, here X^μ and $\gamma_{\alpha\beta}$ (and NOT over the backgrounds themselves!)

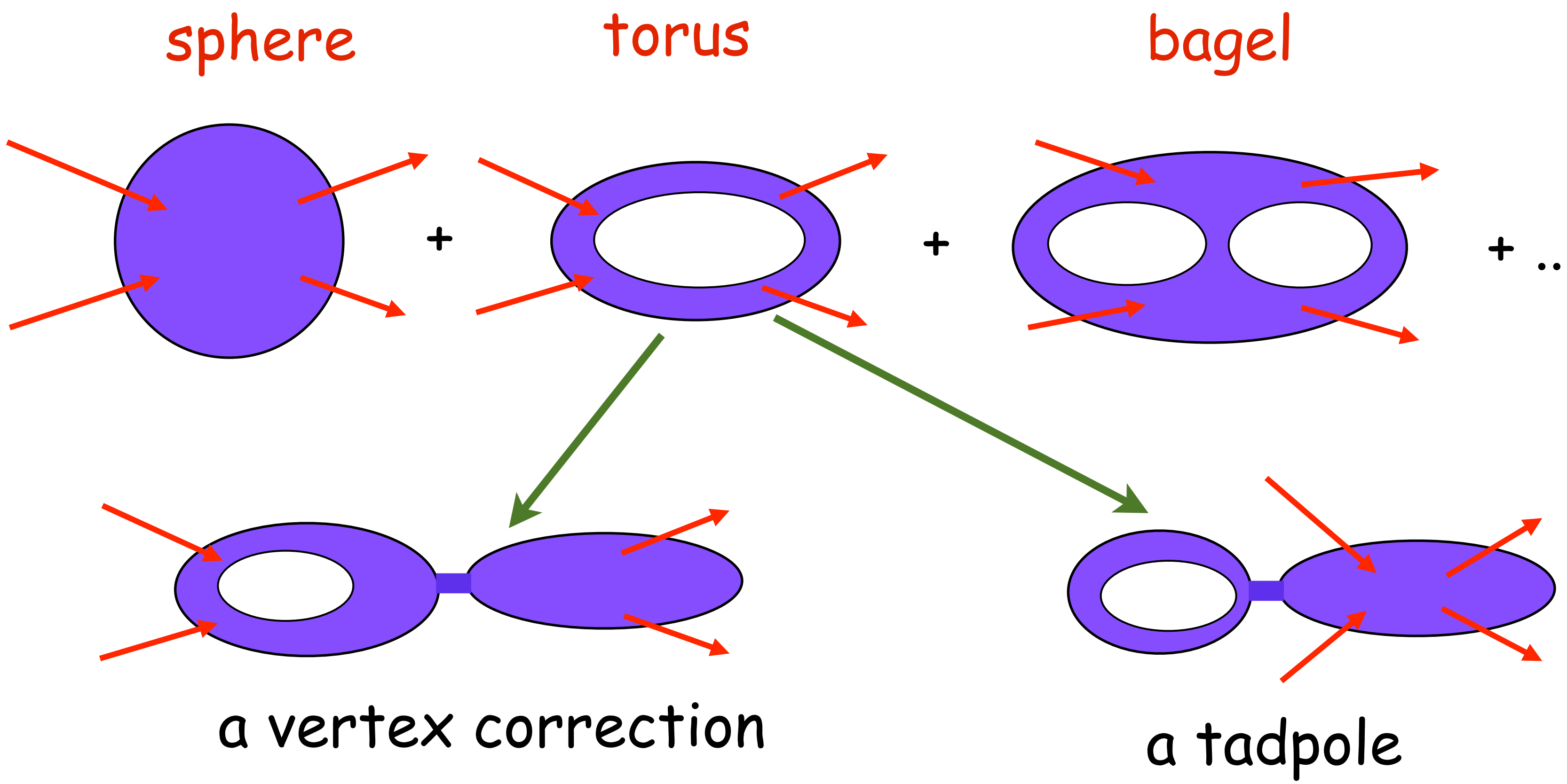
Hence the functional integral over $\gamma_{\alpha\beta}$ splits into a sum of integrals each representing a Riemann surface of a given genus g. Precisely this sum corresponds to the loop expansion in QFT! QST has managed to introduce QFT's loops without invoking any 2nd quantization!

Extra bonus: in QFT the number of diagrams grows like g! Here there is just one diagram at each loop order.

If we also set $B_{\mu\nu} = 0$, $G_{\mu\nu} = \eta_{\mu\nu}$, this is the string we have been discussing so far with just one small additional point.

The contribution of a Riemann surface of genus g to the path integral comes with a factor $\exp(-2\Phi(1-g))$ hence with an extra factor $\exp(2\Phi)$ for each extra string loop. Therefore $\exp(2\Phi)$ plays, in QST, the same role that α plays in QED (or in the SM). It is the loop-counting parameter.

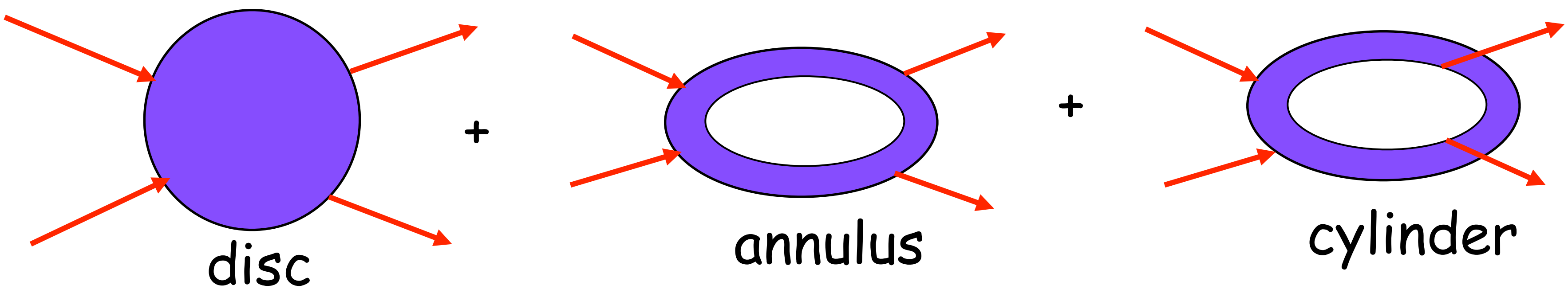
Loop expansion for closed string collisions



Closed strings attach at points on the Riemann surface. These are just our good old Koba-Nielsen variables z_i (complex numbers for closed strings) on which one has to integrate.

Open strings instead attach to boundaries of the Riemann surface, the analogue of quark loops in QCD. The sum over topologies is also a sum over different “boundaries”, their total number, which have strings attached to them and which do not, etc.

The tree level corresponds now to the disc. At one loop we find the annulus, the cylinder, the Moebius strip. One can then also add “handles” (increasing the genus) as for closed strings.



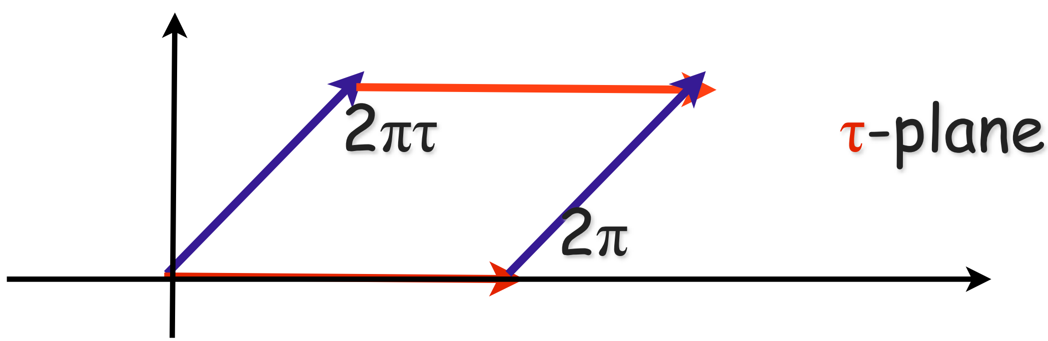
The positions at which the open strings are attached are real, ordered Koba-Nielsen variables on which one has to integrate.

Modular Invariance

Things are even more complicated. For a given topology of the Riemann surface, one has to find out exactly what the integration variables are after gauge fixing. The result is that:

1. For the sphere (and the disc for open strings) there is no integral over the size of the sphere. Furthermore, there is a residual invariance under projective $O(2,1)$ transformations that allows to fix 3 KN coordinates (exactly what we had in the DRM!).

2. For $g=1$ (torus) there is still an integration over the complex parameter τ that characterizes each torus.



3. For $g > 1$ there is an integration over $3(g-1)$ complex parameters that characterize the Riemann surface.

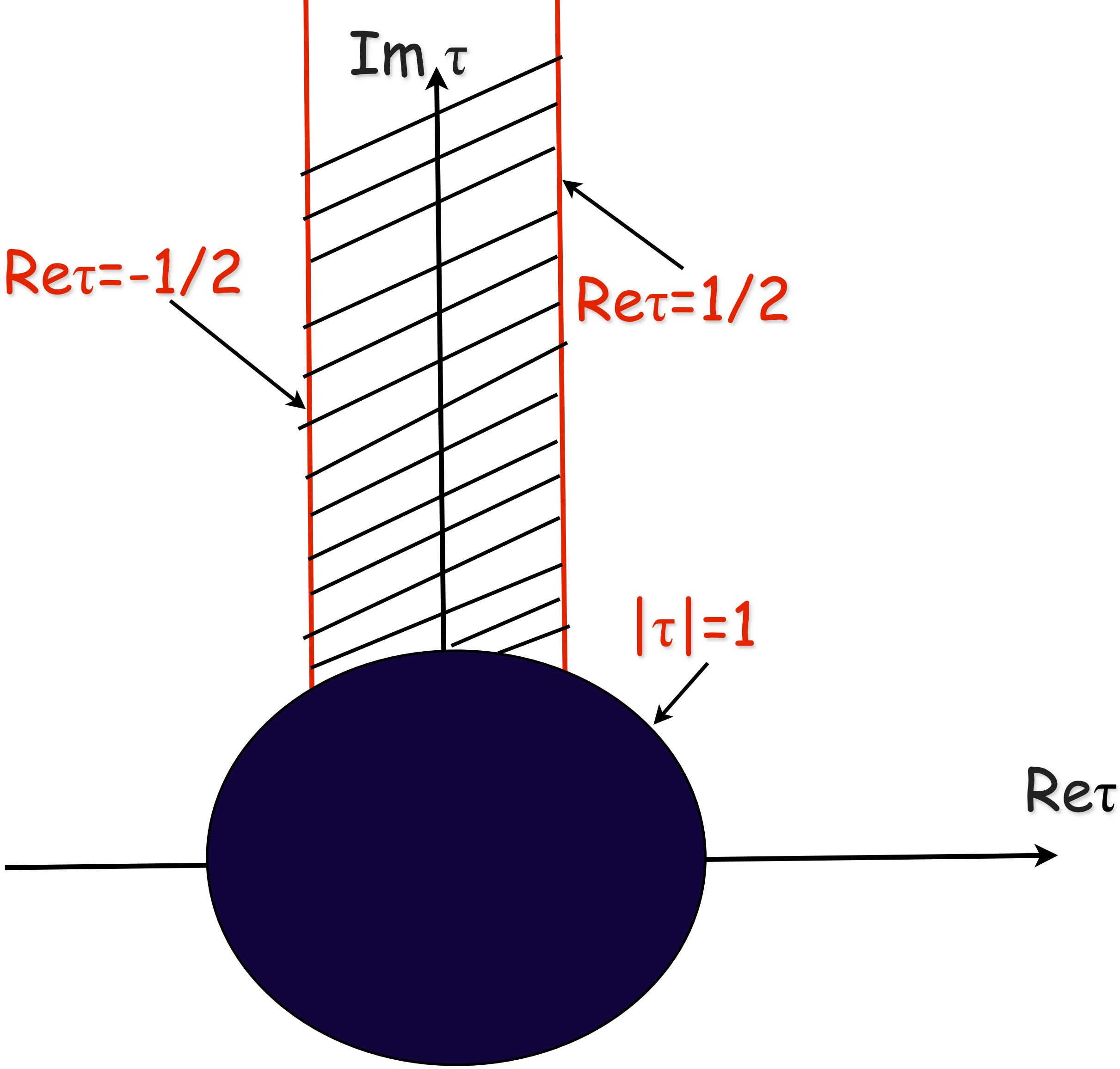
Coming back to the torus, there is still a discrete group of transformations that leaves the torus invariant. This is the group of modular transformations:

$$\tau \rightarrow \frac{p\,\tau + q}{r\,\tau + s} \quad ; \quad p, q, r, s \in Z \quad ; \quad ps - qr = 1$$

It maps the same torus in the complex-τ plane an infinite number of times leading to an infinite result if we were to integrate over the whole complex plane. We should only take one region e.g. the so-called fundamental region. This nicely avoids the point $\tau = 0$ that turns out to be associated to the UV region.

This is the way string theory avoids UV infinities!

It amounts to a cut-off of order $M_s = l_s^{-1}$.



Fundamental region for the torus (shaded)

Modular invariance is as essential for the consistency of string theory as Weyl and reparametrization invariance (they are all parts of the gauge invariances of ST). As it turns out, imposing modular invariance at the one-loop level eliminates the gauge and gravitational anomalies (also one-loop effects!) that the GS mechanism cancels by a brute-force calculation.

The search for consistent QSTs is therefore reduced to the problem of finding theories that respect modular invariance.

This is how the two consistent heterotic string theories HetO (gauge group SO(32)), HetE ($E_8 \times E_8$), were found...

Let's go back to strings in non-trivial backgrounds.

Let's put all 3 terms together and write the action for a string in a metric, antisymmetric and dilaton background as:

$$S = -\frac{T}{2}\int d^2\xi\sqrt{-\gamma}\left[\partial_\alpha X^\mu \partial_\beta X^\nu \left(\gamma^{\alpha\beta}G_{\mu\nu} + \frac{\epsilon^{\alpha\beta}}{\sqrt{-\gamma}}B_{\mu\nu}\right) - \frac{1}{2\pi T}R(\gamma)\Phi\right]$$

Under what conditions for the background fields can we satisfy the conditions of 2D-rep. and Weyl invariance at the quantum level?

This is, in general, a highly non trivial problem. We know one solution: Minkowski spacetime, vanishing B, and constant Φ, provided D takes a critical value (D=26, 10).

In order to look for more general solutions we have to resort to some kind of perturbation theory around the "trivial" backgrounds.

We can do that by expanding $G(X)$ and $B(X)$ around a particular point x. This generates terms in the action that are cubic, quartic and so on in the string coordinates.

From the point of view of a 2-dimensional field theory we go from a free theory to an interacting one where the effective coupling is l_s/L, with L the typical length scale of the geometry (scale over which the backgrounds change by $O(1)$). New contributions to the anomaly (or to the anomaly-cancellation conditions) will come as a power expansion in $(l_s/L)^2 \sim \alpha'$. This is referred to as the α' expansion.

The possible breaking of Weyl invariance can be formulated in terms of 2D β-functions in analogy with what we do when we describe the breaking of scale invariance in QFT in terms of some β- functions of its various couplings.

Since in QST the backgrounds, G, B etc. play the role of couplings there is a β-function(al) associated with each background field.

Setting all β-functions to 0 will give the conditions to be satisfied by the backgrounds in order to preserve all the crucial local 2D symmetries.

In each one of these backgrounds string quantization should be free of pathologies (although it could be quite non-trivial).

A non trivial calculation leads to the following equations to leading non-trivial order in α' (l_s^2):

$$
\begin{aligned}
\beta^{\Phi} &= \frac{D-D_c}{3} + l_s^2\left(\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}D_\mu D^\mu\Phi - \frac{1}{24}H_{\mu\nu\rho}H^{\mu\nu\rho}\right) + O(l_s^4) = 0 \\
\beta^{G}_{\mu\nu} &= l_s^2\left(R_{\mu\nu} + \frac{1}{4}H_{\mu\rho\sigma}H_{\nu}^{\ \rho\sigma} - 2D_\mu D_\nu\Phi\right) + O(l_s^4) = 0 \\
\beta^{B}_{\mu\nu} &= l_s^2\left(D^\rho H_{\mu\nu\rho} - 2\partial^\rho\Phi H_{\mu\nu\rho}\right) + O(l_s^4) = 0 \quad ; \quad H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \text{cyclic}
\end{aligned}
$$

We can now reinterpret in a more satisfactory way the meaning of $D=D_c$(=26, 10). If $D\neq D_c$, there is no solution to the above equations with nearly constant backgrounds. All solutions will have necessarily some fields whose space-time variations are so large to compensate for the extra factor l_s^2. However, in that case, we are not allowed, in principle, to neglect the higher-order corrections (e.g. $O(l_s^4)$) and we cannot be sure, in general, that we do have a solution.

The effective action of QST

A very interesting property of our β-function equations is that they define what is called a "gradient flow": the β-functions are derivatives of a function(al).

That means that they correspond to the eom that follow from an effective action (i.e. by setting to zero its variation wrt the various fields). Up to the order we have considered the effective action reads:

$$\Gamma_{eff} = -\left(\frac{1}{l_s}\right)^{D-2} \int d^D x \sqrt{-G} e^{-2\Phi} \left[\frac{4(D-D_c)}{3l_s^2} + R(G) - 4\partial_\mu \Phi \partial^\mu \Phi + \frac{1}{12} H^2 + \dots \right]$$

1. For the dots, see below.

2. The dilaton appears with the "wrong" sign, but actually there is nothing wrong with this.

Some interesting properties of Γ_{eff}

1. The dilaton appears through an overall factor multiplying something that can only depend on its derivatives. This is as expected since, if Φ is constant, the only dependence on Φ must be an overall factor exp $(-2\Phi(1-g))$.
2. Γ_{eff} contains no arbitrary dimensionless parameters and just one dimensionful one, l_s.
3. Γ_{eff} is general covariant. It is also invariant under $B \dashrightarrow B + d\Lambda$. Indeed, B only enters through its field strength $H = dB$.
4. We will come back to some new (stringy) symmetries of Γ_{eff}.

The two meanings of Γ_{eff}

The effective action actually has two distinct meanings. The first is the one we have just said: it generates (as eom) the conditions to be satisfied by the background fields in order to preserve the 2D local symmetries of string theory.

The second meaning is a more familiar one for an effective action: Γ_{eff} can be used to compute the couplings of various massless particles and their scattering amplitudes as an expansion in powers of energy (Cf. zero-slope limit).

It is amazing that these two concepts get related in string theory.

A theory of gravity but not Einstein's!

In D dimensions, the analogue of the Einstein-Hilbert action takes the form:

$$\frac{1}{\hbar} S_{EH} = \left(\frac{1}{l_P}\right)^{D-2} \int d^D x \sqrt{-g(x)} \left(\Lambda - \frac{1}{2} R(g)\right) \quad ; \quad 8\pi G_N \hbar \equiv l_P^{D-2}$$

while in QST we found:

$$\Gamma_{eff} = -\left(\frac{1}{l_s}\right)^{D-2} \int d^D x \sqrt{-G} e^{-2\Phi} \left[\frac{4(D-D_c)}{3 l_s^2} + R(G) - 4\partial_\mu \Phi \partial^\mu \Phi + \frac{1}{12} H^2 + \dots\right]$$

Are they equivalent up to some field redefinition? The answer is obviously no, even if we set H=0. QST gives a scalar-tensor theory of a Jordan-Brans-Dicke kind!

$$\frac{1}{\hbar}S_{EH} = \left(\frac{1}{l_P}\right)^{D-2}\int d^Dx\sqrt{-g(x)}\left(\Lambda - \frac{1}{2}R(g)\right) \quad ; \quad 8\pi G_N\hbar \equiv l_P^{D-2}$$

$$\Gamma_{eff} = -\left(\frac{1}{l_s}\right)^{D-2}\int d^Dx\sqrt{-G}e^{-2\Phi}\left[\frac{4(D-D_c)}{3l_s^2} + R(G) - 4\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{12}H^2 + \ldots\right]$$

For a constant Φ we can identify l_P^{D-2} with $\exp(2\Phi)\, l_s^{D-2}$ but a massless dilaton still produces long-range interactions that violate the equivalence principle: the dilaton, having spin zero, couples (non universally!) to mass rather than to energy and produces violations of UFF (T. Taylor and GV).

This is a real threat to QST, making it vulnerable even to long-distance/low-energy experiments. In fact, at tree-level, string theory is already ruled out by present precision tests of the EP.

The two expansions of Γ_{eff}

We have (roughly) seen how quantization of (integrating over) the string coordinates produces potential anomalies that have a natural expansion in powers of l_s.

We have also seen that integrating over the 2D metric produces another expansion in powers of $\exp(2\Phi)$.

Therefore Γ_{eff} has a double perturbative expansion:

$$\begin{aligned}\Gamma_{eff} &= -\left(\frac{1}{l_s}\right)^{D-2}\int d^D x\sqrt{-G}e^{-2\Phi}\left[\frac{4(D-D_c)}{3l_s^2}+R(G)-4\partial_\mu\Phi\partial^\mu\Phi+\frac{1}{12}H^2+O(l_s^2)\right]\\ &+ \left(\frac{1}{l_s}\right)^{D-2}\int d^D x\sqrt{-G}\,[\ldots]+O(e^{2\Phi})\end{aligned}$$

One expansion has a QFT analogue. The other does not!

This effective action modifies gravity at large distances (which is dangerous but hopefully cured by loop and non-perturbative corrections) and also, of course, at short distances $O(l_s)$.

These latter modifications make loop corrections well defined in the UV. Indeed, one gets their correct order of magnitude, $\exp(2\Phi)$, by computing loops as in a QFT but with a short distance cutoff given by the string length.

Here is an example of a quantum-gravity loop correction:

$$\left(\frac{\text{loop}}{\text{tree}}\right) \sim G_N \Lambda_{UV}^{D-2} \rightarrow \left(\frac{l_P}{l_s}\right)^{D-2} = exp(2\Phi)$$

which is roughly of the same order as a gauge-loop correction.