Fédération de recherche André Marie Ampère

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# De la corde hadronique à la gravitation quantique... et retour

## Cours V : 12 mars

**Cordes, Branes et Théories de Jauge**

Université Claude Bernard Lyon 1

UNIVERSITÉ DE LYON

ECOLE NORMALE SUPERIEURE DE LYON

# Target-space symmetries in QST

We have already seen that the effective action satisfies some local spacetime symmetries. In particular it is invariant under GCT in spacetime (the principle underlying GR) and under gauge transformations of the B field.

These symmetries are common to QFT and QST, except that in QFT they are imposed from the start while in QST they "emerge" (massless spinning particles are also "emergent").

There are, however, new symmetries in string theory that have no field-theoretic equivalent. Many of them appear to be related to compactification of the extra dimensions.

Gauge symmetries emerge in General Relativity from the Kaluza-Klein (KK) mechanism when some of the spatial dimensions are compact.

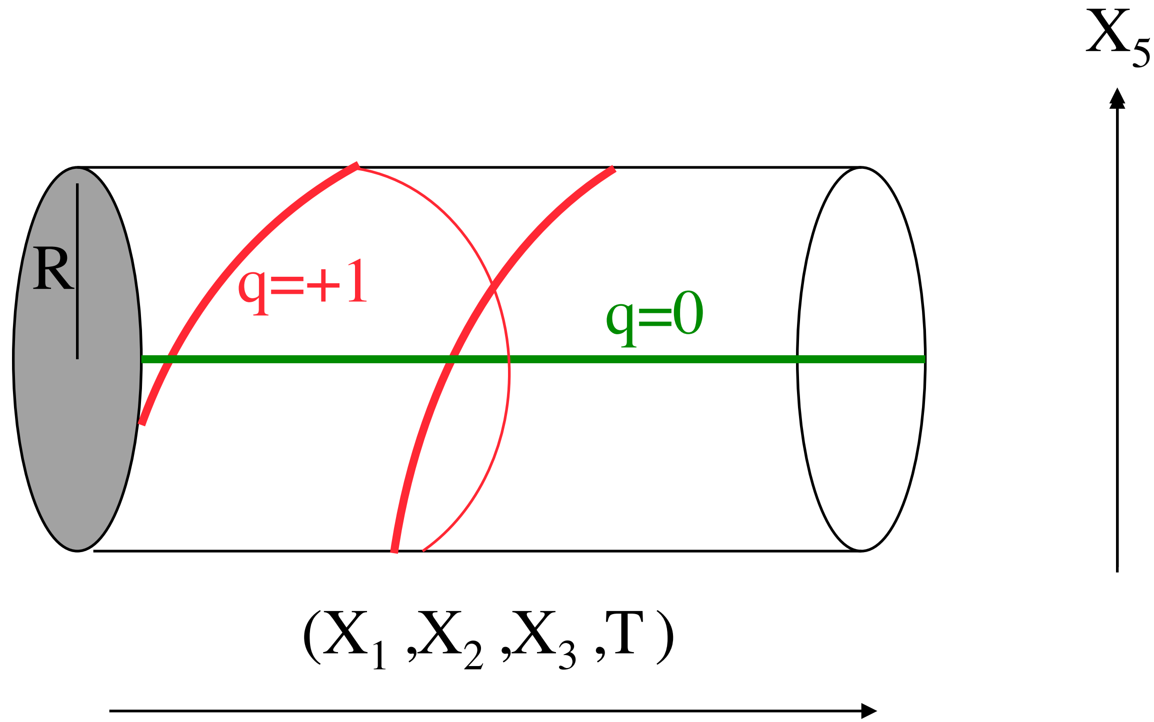
What happens to the KK mechanism in QST?

# A quick reminder of KK theory

Kaluza (1921) and Klein (1926) (KK) managed to reformulate electromagnetism + gravity as just GR in a space containing one extra spatial dimension.

In KK theory the extra dimension of space is a circle of radius R.

The e.m. potential $A_\mu$ becomes the component $g_{\mu 5}$ of the 5-dimensional metric, while $g_{55}$ is a scalar field associated with the (proper) radius of the circle.




$p_5$ is quantized in units of $h/R$ (QM is crucial)

$q = p_5/M_P = n\, l_P/R$ , $n = 0, \pm 1, \pm 2, ..$

Quantization of electric charge automatic!

KK Unification: $F_C = F_N$ at $E \approx hc/R = M_C\, c^2$.

$M_C$ = mass of typical KK excitations.

# The stringy version of KK
## (5 = compact dimension)

In string theory, for a generic value of R, the gauge symmetry is actually U(1)xU(1). The reason is that both $G_{\mu 5}$ and $B_{\mu 5}$ give rise to gauge bosons. General covariance and invariance under gauge transformations of B both become ordinary gauge transformations when the GCT (or B-gauge) parameter is chosen to be independent of $x_5$.

In a QFT context we can add by hand a B-field and get the extra U(1) gauge invariance.

However, while for the U(1) coming from $G_{\mu 5}$ we can identify the charge with the $p_5$, for the U(1) of $B_{\mu 5}$ we cannot find a 5-dimensional meaning for its associated charge.

In string theory (where the B-field is inevitable) we have instead a very elegant interpretation.

Recall the boundary conditions for the closed bosonic string:

$$X'_\mu \delta X^\mu(\sigma = 0) = X'_\mu \delta X^\mu(\sigma = \pi) \quad ; \quad (\text{no sum over } \mu)$$

Strict periodicity of $X_5$ clearly satisfies the b.c. However, since now the points $X_5$ and $X_5 + 2\pi$ m R are identified, there is nothing wrong with imposing, instead,

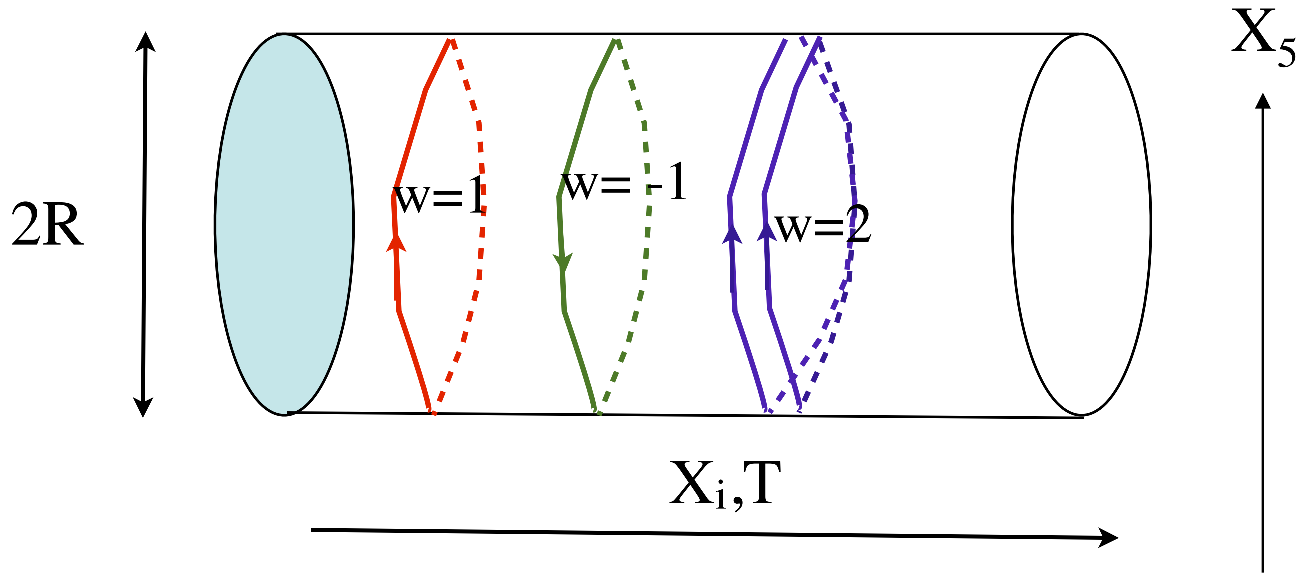
$X_5\,(\sigma=\pi) = X_5\,(\sigma=0) + 2\pi$ w R with w an integer.

It simply means that the closed string winds around the compact direction w-times! Note that winding is a topological property of closed strings which has no analogue for points or open strings.

And, of course, winding costs length, hence energy because of the string tension.

The new boundary condition is easily implemented in the general solution by adding a "winding term".

NB: point particles (and open strings?) cannot wind!




$$
\begin{aligned}
X_5(\sigma,\tau) &= q_5 + 2n\alpha'\frac{\hbar}{R}\tau + 2wR\sigma \\
&+ \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-2in(\tau-\sigma)} - \frac{a^{\dagger}_{n,5}}{\sqrt{n}}e^{2in(\tau-\sigma)}\right] \\
&+ \frac{i}{2}\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{\tilde{a}_{n,5}}{\sqrt{n}}e^{-2in(\tau+\sigma)} - \frac{\tilde{a}^{\dagger}_{n,5}}{\sqrt{n}}e^{2in(\tau+\sigma)}\right]
\end{aligned}
$$

The "charge" for the $G_{\mu 5}$ gauge boson, is $p_5$!

The "charge" for the $B_{\mu 5}$ gauge boson, is winding!

The closed bosonic string mass shell conditions are:

$$L_0 = 1 \quad \Rightarrow \quad M^2 = \left(\frac{\hbar n}{R} + \frac{wR}{\alpha'}\right)^2 + \frac{4}{\alpha'}(N-1)$$

$$\tilde{L}_0 = 1 \quad \Rightarrow \quad M^2 = \left(\frac{\hbar n}{R} - \frac{wR}{\alpha'}\right)^2 + \frac{4}{\alpha'}(\tilde{N}-1)$$

$$M^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2) \; ; \; N - \tilde{N} + nw = 0$$

Let us find which states have zero mass. For a generic R it is quite clear that the only zero-mass states are given by:

$$N = \tilde{N} = 1 \; ; \; n = w = 0 \; ; \; \text{i.e.} \quad a^\dagger_{1\mu} \tilde{a}^\dagger_{1\nu} |0\rangle$$

In the absence of compactification these are just $(D-2)^2$ physical states: a graviton, a dilaton, an antisymmetric tensor. After compactification these $(D-2)^2$ massless states split into a graviton, a dilaton, an antisymmetric tensor in (D-1) dimensions (giving $(D-3)^2$ states), two (D-3)-component vectors, and 1 scalar, the “radion”.

Something quite remarkable happens, however, if we choose a particular value for R:

$$R = R^* \equiv \sqrt{\hbar\alpha'} = \frac{l_s}{\sqrt{2}}$$

In this case there are ways of getting massless strings on top of those we had for generic R:

$$n = w = \pm 1 \ ; \ N = 0, \tilde{N} = 1 \quad \text{or}$$
$$n = -w = \pm 1 \ ; \ N = 1, \tilde{N} = 0$$

These are 4 massless vectors, two left and two right-moving. Together with the 2 previous ones they are the 6 gauge bosons of an SU(2)xSU(2) gauge group w/ the two factors corresponding to left and right-moving states. Note that the 4 new gauge bosons carry momentum and winding and are therefore themselves charged, a characteristic of non-abelian gauge theories.

The above solutions also provide 4 massless scalars (when we take the oscillator in the 5th direction).

Actually there are 4 more massless scalars corresponding to taking the oscillator vacuum and ($n = \pm 2$, $w=0$) or ($n = 0$, $w = \pm 2$).

The total number of massless scalars is thus 10. Leaving a singlet dilaton aside, they form a (3,3) representation of

SU(2)xSU(2). The radion corresponds to a particular direction in both SU(2)'s and plays the role of a Higgs field that breaks spontaneously SU(2)xSU(2) down to U(1)xU(1) away from the special point $R = R^*$.

The mass of the gauge bosons corresponding to the broken generators is linear in $(R-R^*)$.

But the surprises are not over...

# T-duality (for closed strings)

Winding and momentum appear on a similar footing in the expression for $X_5$ and for $M^2$. At the classical level there is no possible symmetry between winding and momentum since winding number is an integer, while momentum in the compact direction is not quantized.

At the quantum level single-valuedness of the wave function forces momentum to be quantized in units of $h/2\pi R$. Then, suddenly, a symmetry appears between winding and momentum. The Hamiltonian is invariant under the exchange of $n$ and $w$ and, at the same time, of $R$ into $l_s^2/2R$. Note that the point of enhanced gauge symmetry is precisely the fixed point of this "T-duality" transformation!

Yet another miracle of QST!

We conclude that the inequivalent compactifications are labelled by the range R > R* so that, effectively, there is a minimal compactification radius R=R*. Furthermore, precisely at R=R*, a non-abelian symmetry emerges.

There are reasons to believe that, dynamically, string theory has a preference for such a value of R. Indeed, if some non-perturbative dynamics generates a potential for the radion field and T-duality is respected, the potential will have an extremum (minimum?) at the self-dual value of R.

Possibly, there will be two preferred values of R, R = infinity and R = R*. They could correspond, respectively, to the 3 macroscopic dimensions of our space and to the hidden compactified ones...

# T-duality for open strings & D-branes

T-duality can be seen as a canonical transformation exchanging P and X′ which becomes a full quantum symmetry.

It looks so peculiar to closed strings that, for many years, no one thought that anything similar could apply to open strings since, like point-particles, they cannot wind.

However, exchanging P and X′ can also be done for open strings... Something looks wrong (or rather looked wrong to Polchinski in 1995).

It was the start of the so-called 2$^{nd}$ revolution (almost 10 years after the 1984 GS revolution and 20 after the SS proposal).

The key to solving this puzzle is in the boundary conditions for open strings:

$$X'_\mu \delta X^\mu(\sigma = 0) = X'_\mu \delta X^\mu(\sigma = \pi) \quad ; \quad (\text{no sum over } \mu)$$

Neumann boundary conditions correspond to setting $X'=0$ at the ends of the open string, while Dirichlet boundary conditions mean $\delta X=0$, which amounts to setting $P = 0$.

It looks therefore highly reasonable that, for open strings, T-duality simply changes their boundary conditions from N to D, and vice versa.

Unlike closed strings, open string are not "self T-dual": they come in two kinds which are T-dual to each other!

N.B. We can choose N or D boundary conditions independently for each (spatial) string coordinate.

If we set Dirichlet BC for a certain number n of spatial directions, the ends of such strings are only free to move in the remaining (D-n-1) spatial directions which span a hyperplane.

Such hyperplanes (in general hypersurfaces) are called D-branes or, more precisely, Dp-branes, where p = (D-n-1) is the number of spatial dimensions of the hyperplane. To these one should add time to arrive at p+1 = D-n, the dimension of the "world-volume" where the ends of our open D-strings live.

Such open strings have (p+1) Neumann and n Dirichlet directions.

Let's now look at how this works when we compactify one dimension, $x_5$. The $X_5$ coordinate of an N-string is given by:

$$X_5^{(N)}(\sigma,\tau) = q_5 + 2n\alpha' \frac{\hbar}{R}\tau + i\sqrt{2\alpha'} \sum_{n=1}^{\infty} \left[ \frac{a_{n,5}}{\sqrt{n}} e^{-in\tau} - \frac{a_{n,5}^{\dagger}}{\sqrt{n}} e^{in\tau} \right] cos(n\sigma)$$

We now use T-duality i.e. interchange $P_5$ and $TX'_5$. It is easy to see that the result is simply:

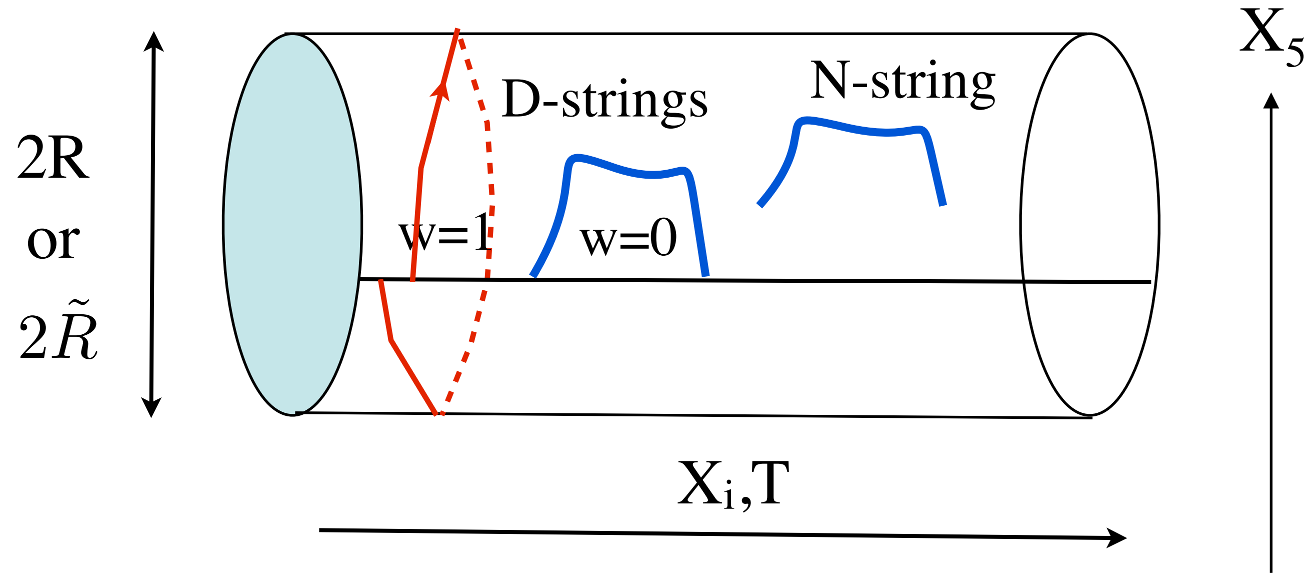
$$X_5^{(D)}(\sigma,\tau) = q_5 + 2w\tilde{R}\sigma + i\sqrt{2\alpha'} \sum_{n=1}^{\infty} \left[ \frac{a_{n,5}}{\sqrt{n}} e^{-in\tau} - \frac{a_{n,5}^{\dagger}}{\sqrt{n}} e^{in\tau} \right] sin(n\sigma)$$

Indeed:

$$\begin{aligned} \frac{1}{T} P_5^{(N)} &= \dot{X}_5^{(N)} = 2n\alpha' \frac{\hbar}{R} + \dots = n\frac{l_s^2}{R} + \dots \\ X_5^{'(D)} &= 2w\tilde{R} + \dots \; ; \text{ OK if } \; \tilde{R} = \frac{l_s^2}{2R} \end{aligned}$$

But then the D-string winds around the dual circle w-times! D-strings can wind!!

# NB: T-dual N and D-strings move/wind around dual circles!

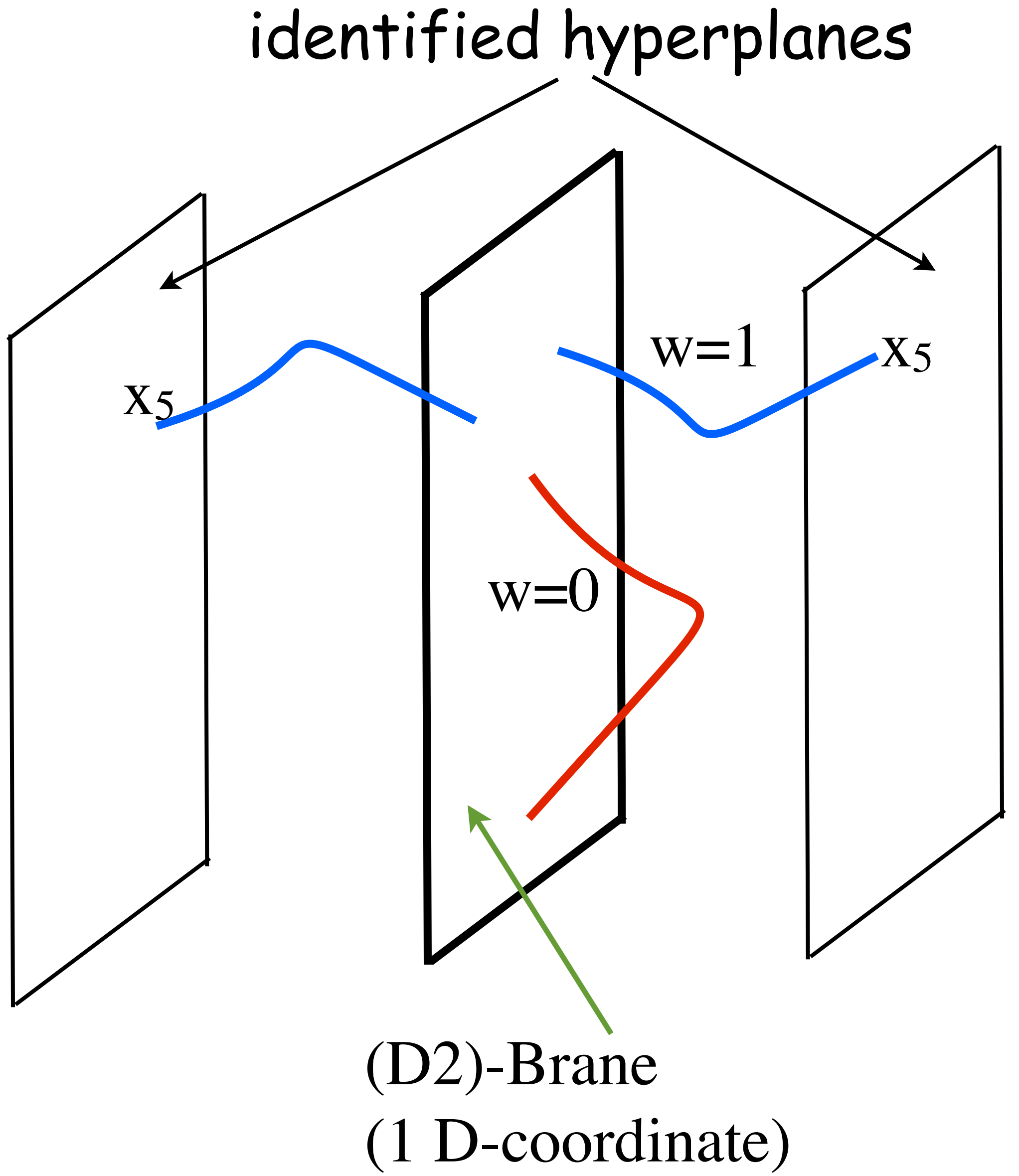



$$X_5^{(N)}(\sigma,\tau) = q_5 + 2n\alpha' \frac{\hbar}{R}\tau + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-in\tau} - \frac{a_{n,5}^{\dagger}}{\sqrt{n}}e^{in\tau}\right] cos(n\sigma)$$

$$X_5^{(D)}(\sigma,\tau) = q_5 + 2w\tilde{R}\sigma + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[\frac{a_{n,5}}{\sqrt{n}}e^{-in\tau} - \frac{a_{n,5}^{\dagger}}{\sqrt{n}}e^{in\tau}\right] sin(n\sigma)$$

$$X_5^{(D)}(\sigma=\pi,\tau) = X_5^{(D)}(\sigma=0,\tau) + 2\pi\tilde{R}\; w$$

# A higher-dimensional example



For the open bosonic string the mass shell condition reads:

$$L_0 = 1 \Rightarrow M^2 = \frac{\hbar^2 n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{1}{\alpha'}(N-1)$$

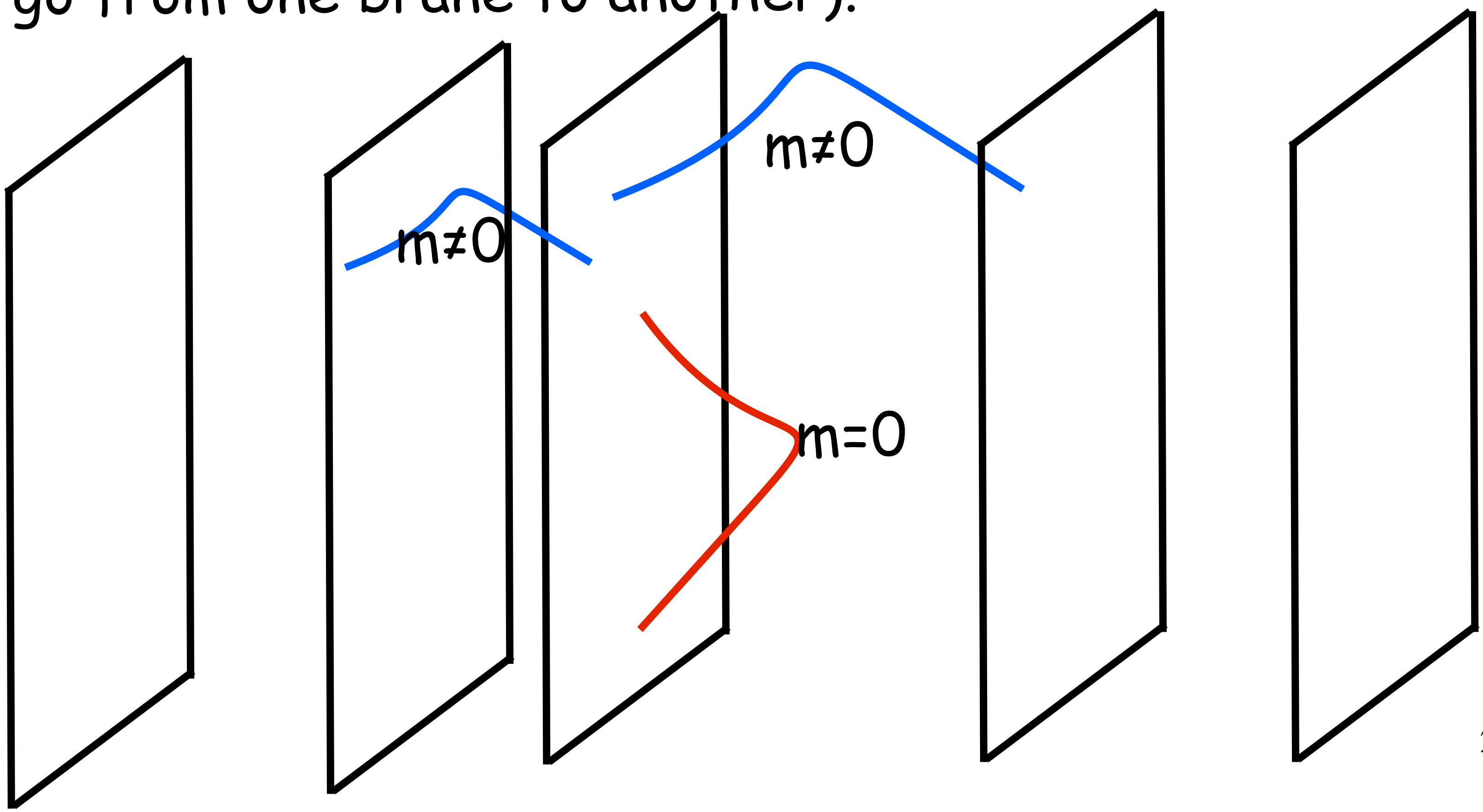
where w=0 for the N-case and n=0 for D. For generic R the massless states are given by n=w=0, N=1, i.e. by the states $a^{\dagger}_{1\mu}|0\rangle$. Let us concentrate on the Dirichlet case.

If the index of the oscillator is not 5 this is a gauge boson stuck on the brane (in (D-1)-dimensions with (D-3) physical components); if the index is 5 it's a massless scalar also confined to the brane. What's the meaning of this scalar?

The answer is quite simple and elegant. The presence of the brane clearly breaks (spontaneously) translation invariance in the 5th direction. The massless scalar is the Nambu-Goldstone boson of that broken symmetry and describes the possible local deformations of the brane itself!

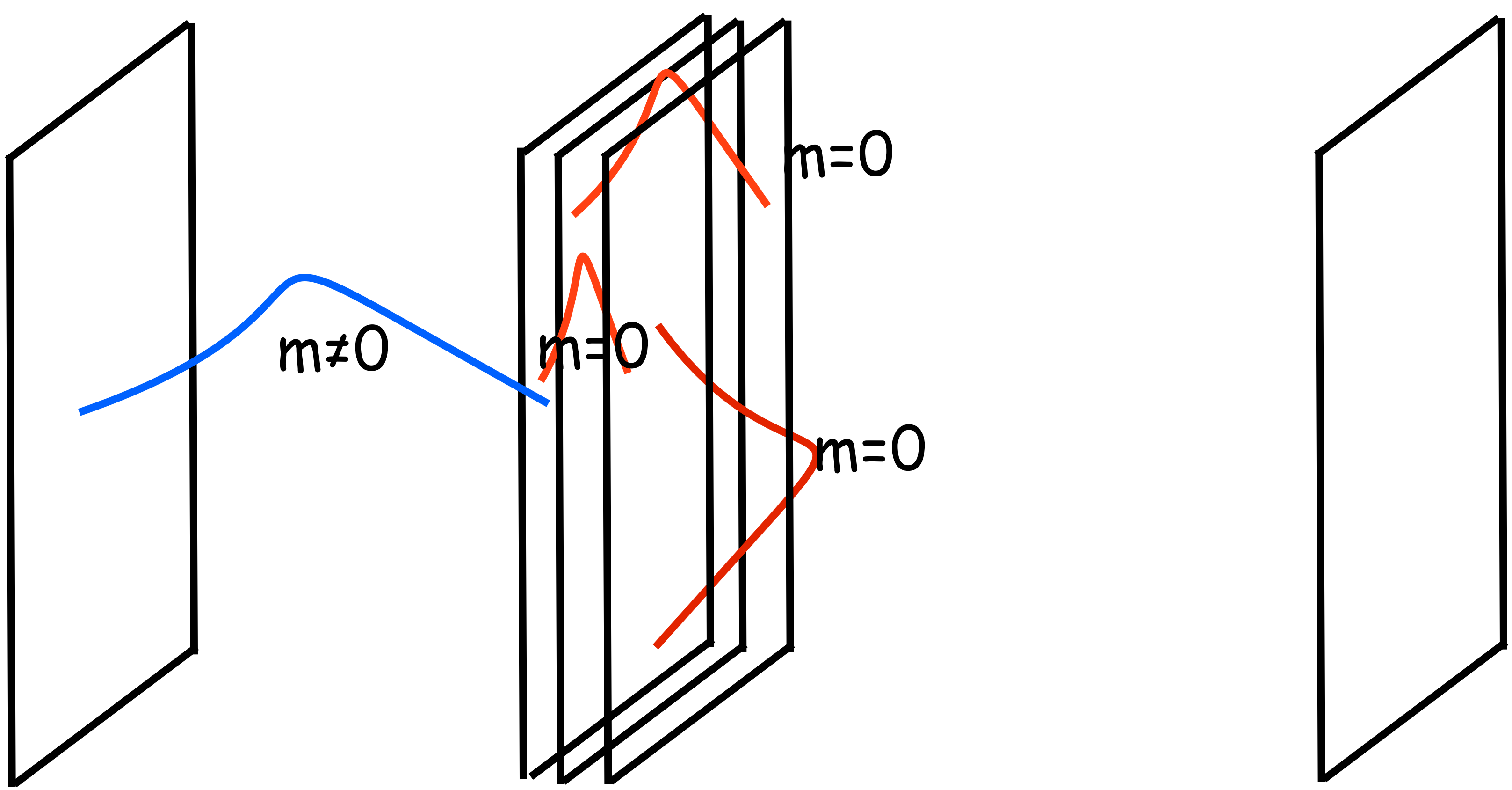
# Multibrane systems (no compactification)

Consider a stack of N parallel D-branes on which open D-strings end. There are $N^2$ kinds of open (oriented) strings depending on which branes they end on. If the two ends are on the same brane the spectrum contains a massless gauge boson. Otherwise there is no massless state (it takes energy to go from one brane to another).



Let's now move n out of the N branes until they overlap. Now the open strings stretched between a pair of these n coincident branes will contain massless gauge bosons as well. The model generates a $U(n)$ gauge theory! If all the N branes coincide we have an unbroken $U(N)$.

In general the symmetry is broken down to $U(n_1)\, U(n_2) \ldots U(n_k)$ and, in the "worst" case, to $U(1)^N$.



Note that in the generic case each "diagonal" massless vector is accompanied by a massless scalar interpreted as the field describing the transverse fluctuations of that brane. The relative distance of two branes plays the role of a Higgs field.

One can study many properties of gauge theories (e.g. Seiberg's duality) from this brane construction viewpoint with interesting new insights.

This is already, in some sense, going back from a theory of strings and gravity to something useful for gauge theories.

However, a qualitatively new development only came out in 1997 when, putting together various hints, Maldacena formulated his now-celebrated conjecture known under the name of AdS/CFT correspondence.

This will be the final subject of our course.

# D-branes as classical solutions

We have described D-branes from an open-string viewpoint (hypersurfaces on which open strings end) but actually D-branes also emerge as classical solutions of the string effective action when we add all the massless bosonic fields contained in Type IIa or IIb superstring theories. Of crucial importance are the RR forms present in such theories since, as it turns out, D-brane are "charged" under those fields (i.e. they are sources for them). A p-brane couples naturally to a (p+1)-form potential. Type IIa, having odd forms, gives rise to even-p-branes, the opposite being the case for Type IIb. To get D=4 QFTs we need IIb.

Another important property of D-branes is that they are very heavy in the small coupling limit. This is why they are not in the perturbative spectrum.Their tension is given by

$$T_p \sim \frac{T}{g_s l_s^{p-1}}$$

The solutions are relatively simple: metric, dilaton and the relevant forms only depend on the transverse distance from the brane. If we take N-coincident D3-branes we find (i=1,2,3):

$$ds^2 = \left(1 + \frac{R^4}{r^4}\right)^{-\frac{1}{2}} (-dt^2 + dx^i dx^i) + \left(1 + \frac{R^4}{r^4}\right)^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2)$$

There is also a non-trivial 4-potential (and 5-field-strength) and a constant arbitrary dilaton providing the string coupling $g_s$. The radius R of this geometry is simply given by the analog of a BH radius in D= 10:

$$R^4 \sim G_{10} N T_3 \sim g_s^2 \alpha' l_s^6 N \frac{T}{g_s l_s^2} \sim g_s N l_s^4 \Rightarrow R \sim (g_s N)^{1/4} l_s$$

NB: The radius of the geometry in string units depends on $g_s$ N

# Taking the relevant limits

$$R^4 \sim G_{10} N T_3 \sim g_s^2 \alpha' l_s^6 N \frac{T}{g_s l_s^2} \sim g_s N l_s^4 \Rightarrow R \sim (g_s N)^{1/4} l_s$$

The interesting limit is one in which N goes to infinity with $\lambda = g_s N$ held fixed. From the point of view of the gauge theory living on the branes this is just the 't Hooft limit since $g_{YM}^2 \sim g_s$. That theory is $N_{susy} = 4$ SYM (a very supersymmetric extension of QCD known to be scale invariant at the quantum level).

On the other hand the Newton constant goes to 0 and graviton loops are negligible. Gravity becomes semiclassical. Depending on the value of $\lambda = g_s N$ the radius of the geometry can be small or large in string units justifying, in the latter case, a SUGRA approximation.

Note that this is the large-coupling limit on the gauge theory side.

# The near-horizon limit

$$ds^2 = \left(1 + \frac{R^4}{r^4}\right)^{-\frac{1}{2}} (-dt^2 + dx^i dx^i) + \left(1 + \frac{R^4}{r^4}\right)^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2)$$

The last limit we want to take is the so-called near-horizon limit $r \ll R$ in which the metric takes the form:

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx^i dx^i) + \frac{R^2}{r^2}(dr^2 + r^2 d\Omega_5^2)$$

Changing variable $r/R = R/\rho$ we get:

$$ds^2 = \frac{R^2}{\rho^2}(-dt^2 + dx^i dx^i + d\rho^2) + R^2 d\Omega_5^2$$

which is the metric of $AdS_5 \times S_5$. This spacetime has a large group of isometries that coincides with the conformal group under which the gauge theory is invariant. This prompted Maldacena to make his conjecture.

# The Maldacena Conjecture

The supergravity theory (or its Type IIb progenitor) lives in D = 10 while the gauge theory lives at r =0, i.e. at $\rho$ = infinity, the boundary of AdS space.

The Maldacena conjecture states that there is a dual correspondence (made mathematically precise by Witten in a subsequent paper) between $N_{susy} = 4$ SYM on the boundary of $AdS_5$ and D=10 SUGRA around its $AdS_5 \times S_5$ solution.

The correspondence should work at large N and at any value of the 't Hooft coupling indentified, on the gravity side, with the appropriate power of $R_{AdS}/l_s$.

On the gauge side one computes correlators of gauge-invariant operators while on the gravity side one computes the analog of some gravitational scattering amplitudes (but in AdS).

One can even study the gauge theory at finite temperature by adding an AdS black hole on the gravity side.

# Checks, predictions, generalizations

By its very nature (mapping hard problems on one side to easy problems on the other) the AdS/CFT correspondence is predictive but not easy to check.

Yet, 15 years after its proposal no contradictions have emerged and a few checks have been possible (particularly through the progress made on solving the SYM side). By now basically nobody doubts its validity.

The remaining problems concern extending the correspondence to more realistic gauge theories such as QCD. Nice attempts have been proposed, but describing the duals of asymptotically free gauge theories meets with big problems. Also going beyond the SUGRA approximation is a technical challenge.

To the extend that high-temperature QCD in the strongly coupled plasma regime can be approximated by N=4 SYM, AdS/CFT predicts a small viscosity/entropy ratio in apparent agreement with experimental indications.

The number of papers on gravity/gauge duality has literally exploded and there is no time to give here even a very short account of them.

Other rigorous correspondences have been proposed involving other dimensions on the gauge theory side and other geometries on the gravity side.

Possible connections to condensed matter systems have also been proposed.

It would also be nice to use the correspondence in the other direction (which I personally find even more interesting): use known properties of gauge theories to find what happens to quantum (string) gravity in the high curvature regime.

The already mentioned applications to the information paradox are already an interesting step in this direction.

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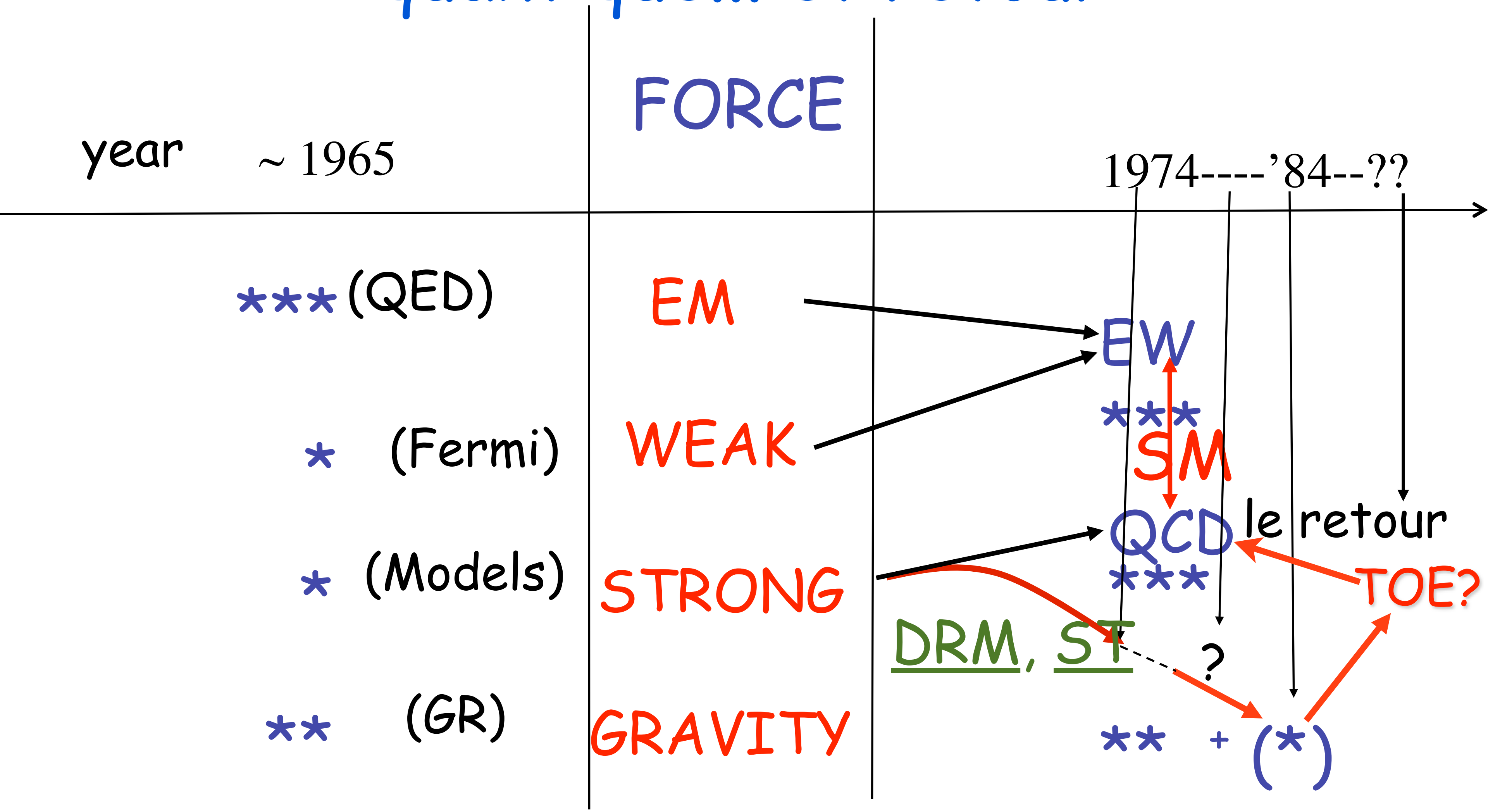

MERCI!